Chapter 3
The Earth's dipole field

Previously

- The Earth is associated with the geomagnetic field that has an S-pole of a magnet near the geographic north pole and an N-pole of a magnet near the geographic south pole.
- A magnetic compass, therefore, approximately points toward the north.
  - The pointing direction is slightly different from the true north by an angle called "declination".
  - A magnetic needle suspended at a center of balance does not keep horizontal. As a rule, the N-pole dips downward by an angle called "inclination" in the northern hemisphere.

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- Geomagnetic poles
- Geomagnetic equator
- Virtual geomagnetic poles
- Geomagnetic coordinates
- Geomagnetic time
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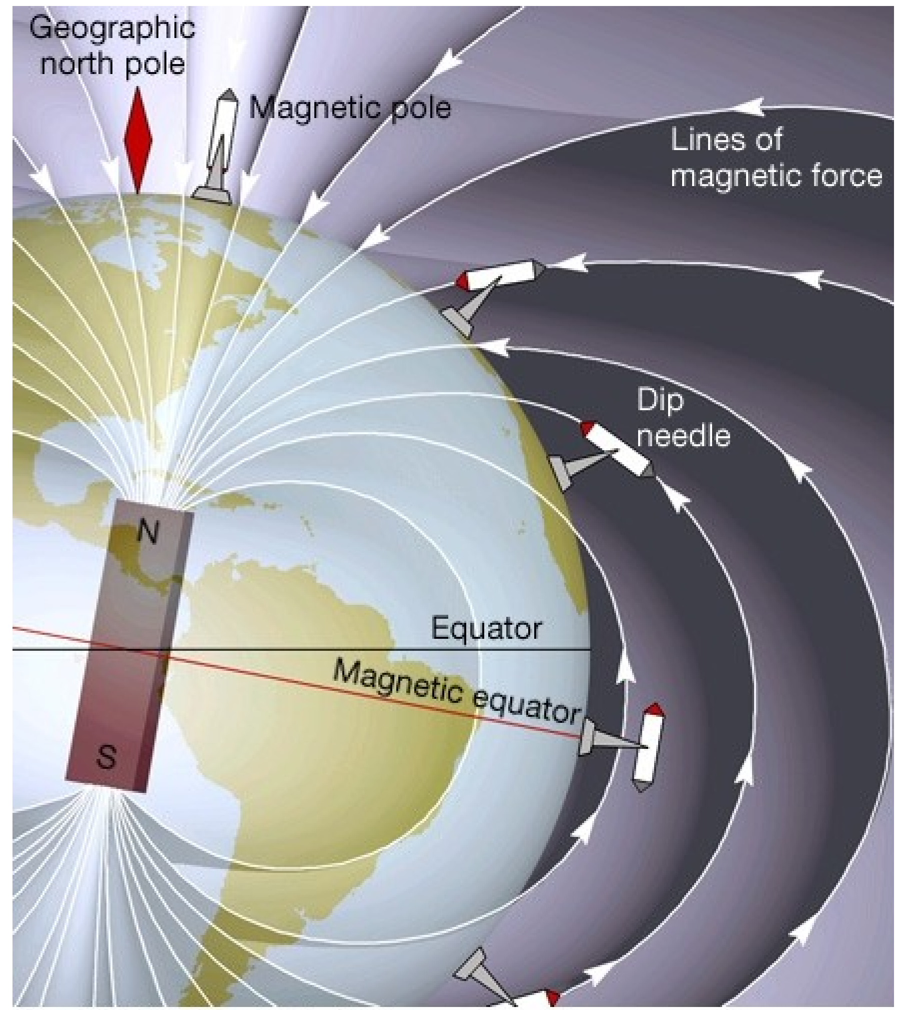
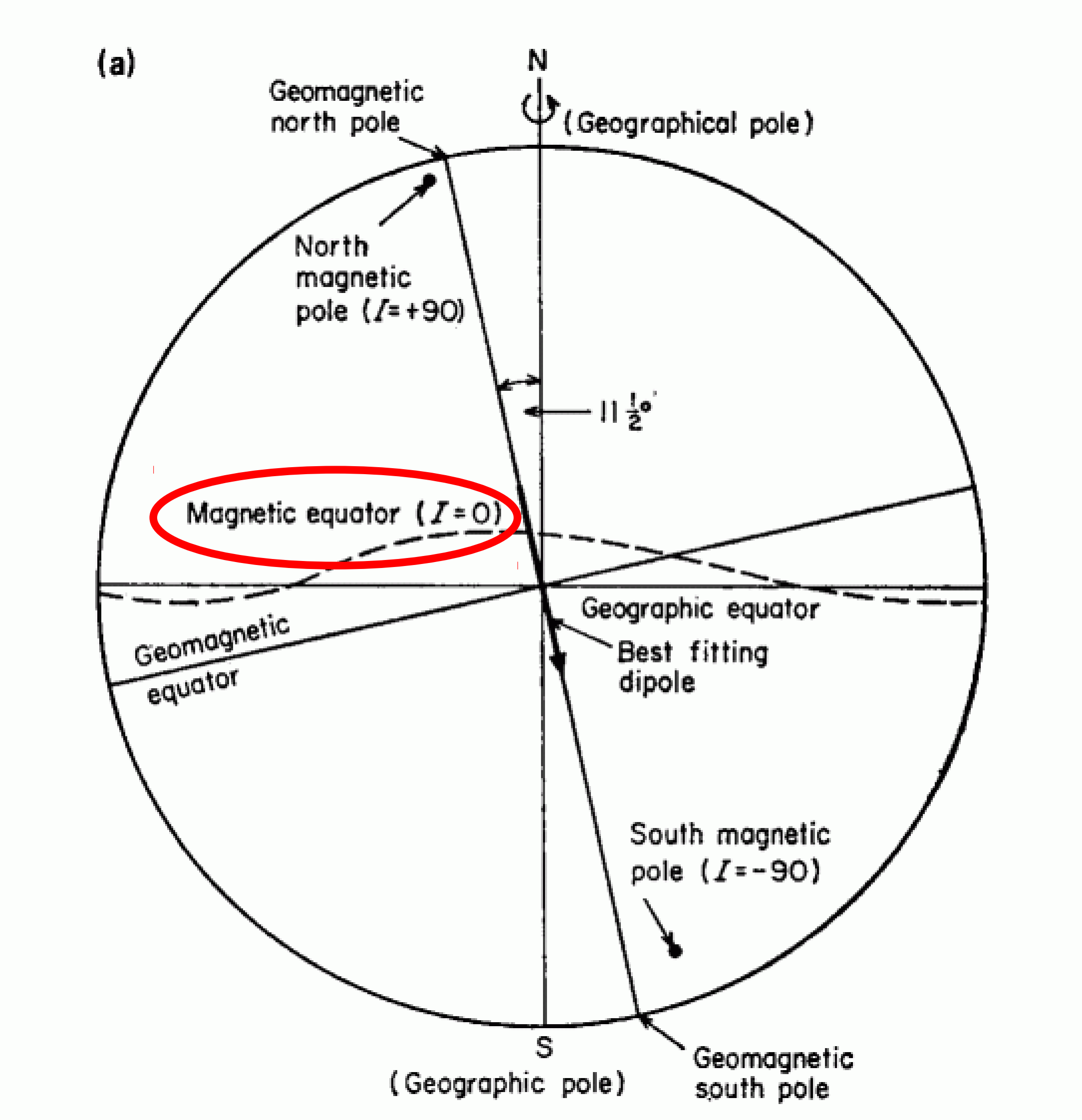
Magnetic poles

- Positions on the Earth's surface where the horizontal component of the magnetic field is zero (the field is vertical).
- These are called dip poles.
- The north and south dip poles do not have to be (and are not now) antipodal.
  - North magnetic pole in the northern hemisphere: $H = 0$ and inclination $I = +90^{\circ}$
  - South magnetic pole in the southern hemisphere: $H = 0$ and $I = -90^{\circ}$
- In principle, the dip poles can be found by experiment, conducting a magnetic survey to determine where the field is vertical.
- Around a magnetic pole, declination D has all possible values ($-180^{\circ} \ldots 180^{\circ}$). Another such a singular point is a geographic pole.
- The locations of the magnetic poles change with time.

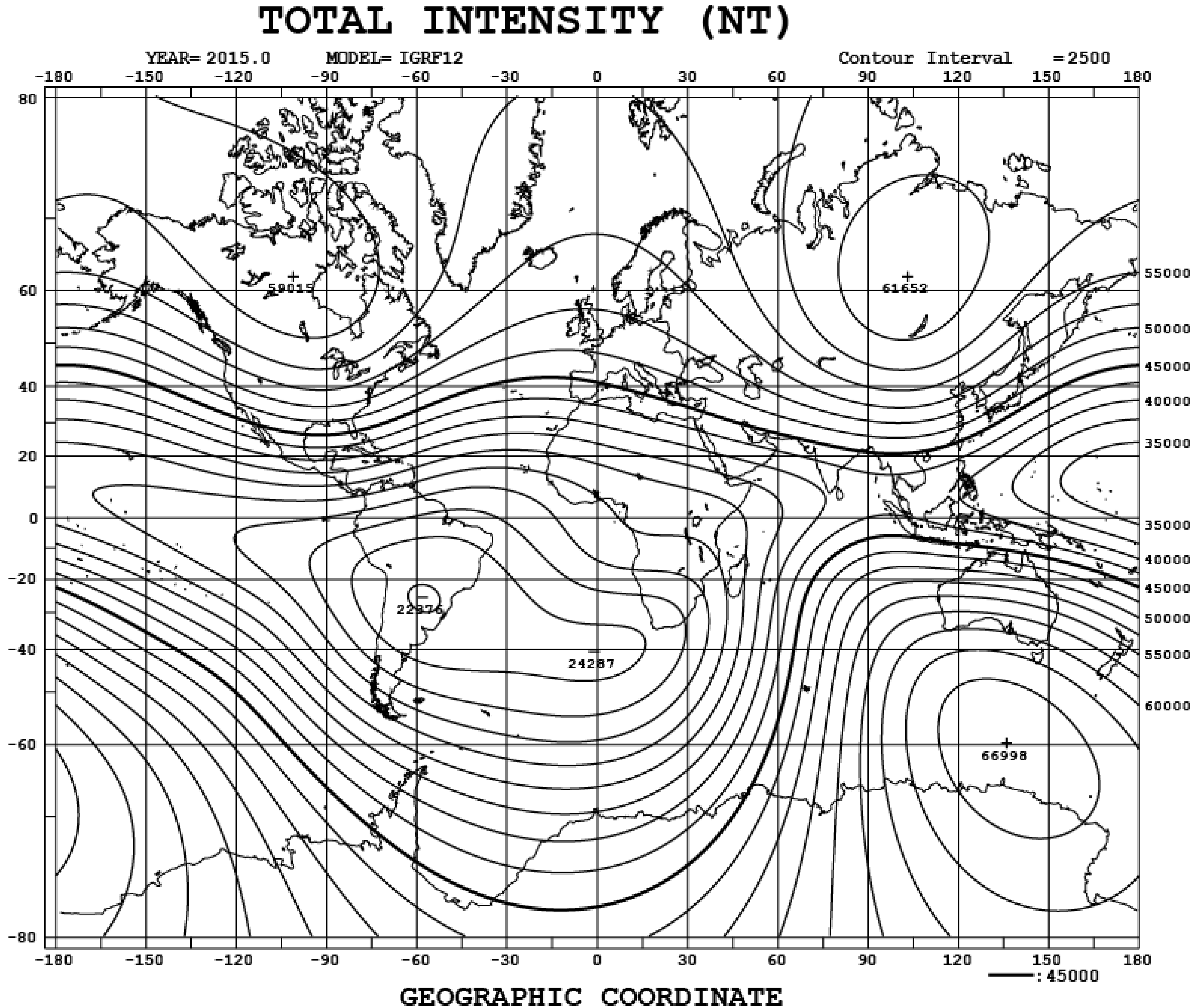


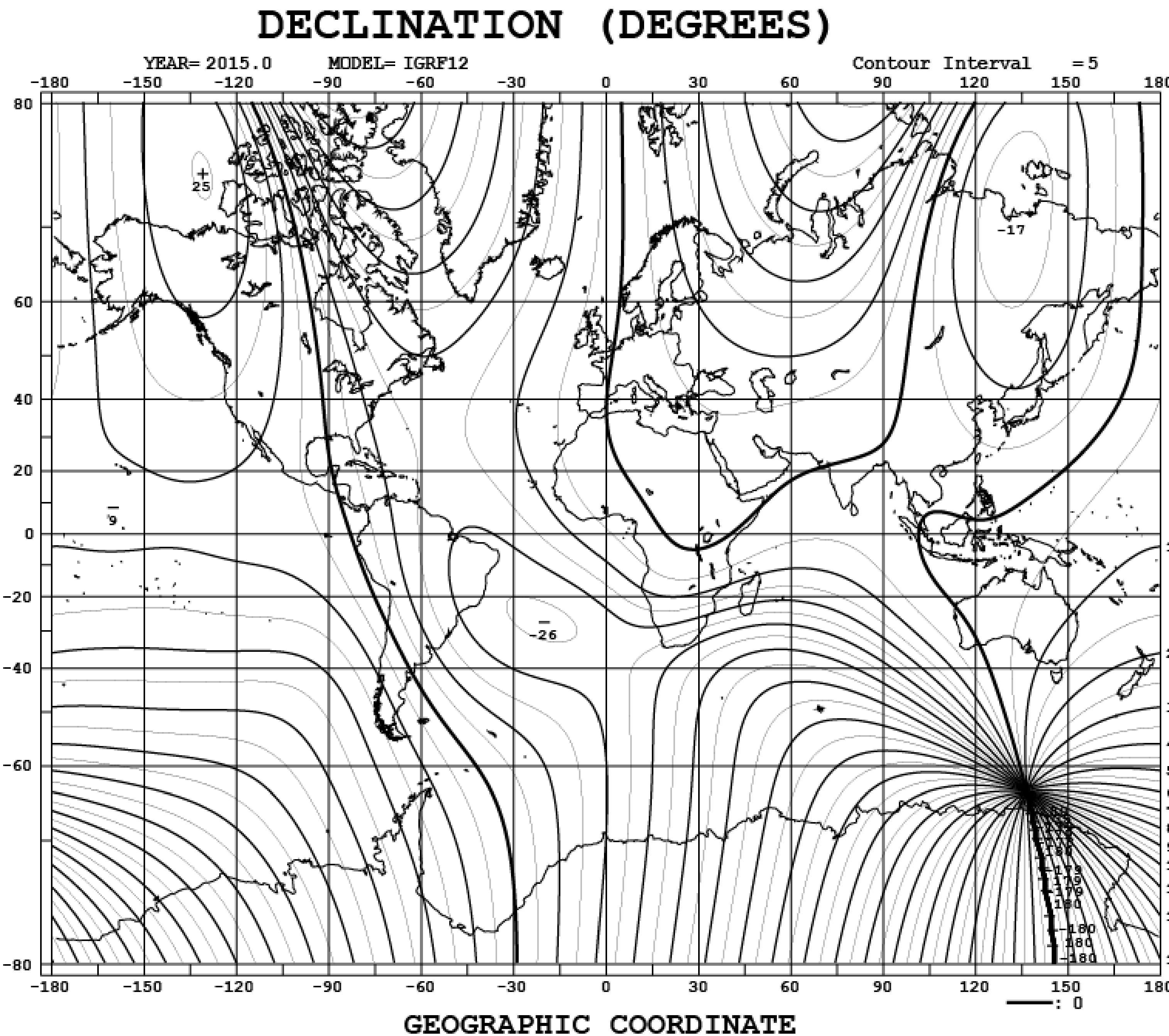
Magnetic equator

- An undulating curve near the Earth's geographic equator where the vertical component of the magnetic field is zero ($Z = 0$ and $I = 0^{\circ}$).



http://wdc.kugi.kyoto-u.ac.jp/igrf/




DECLINATION (DEGREES)
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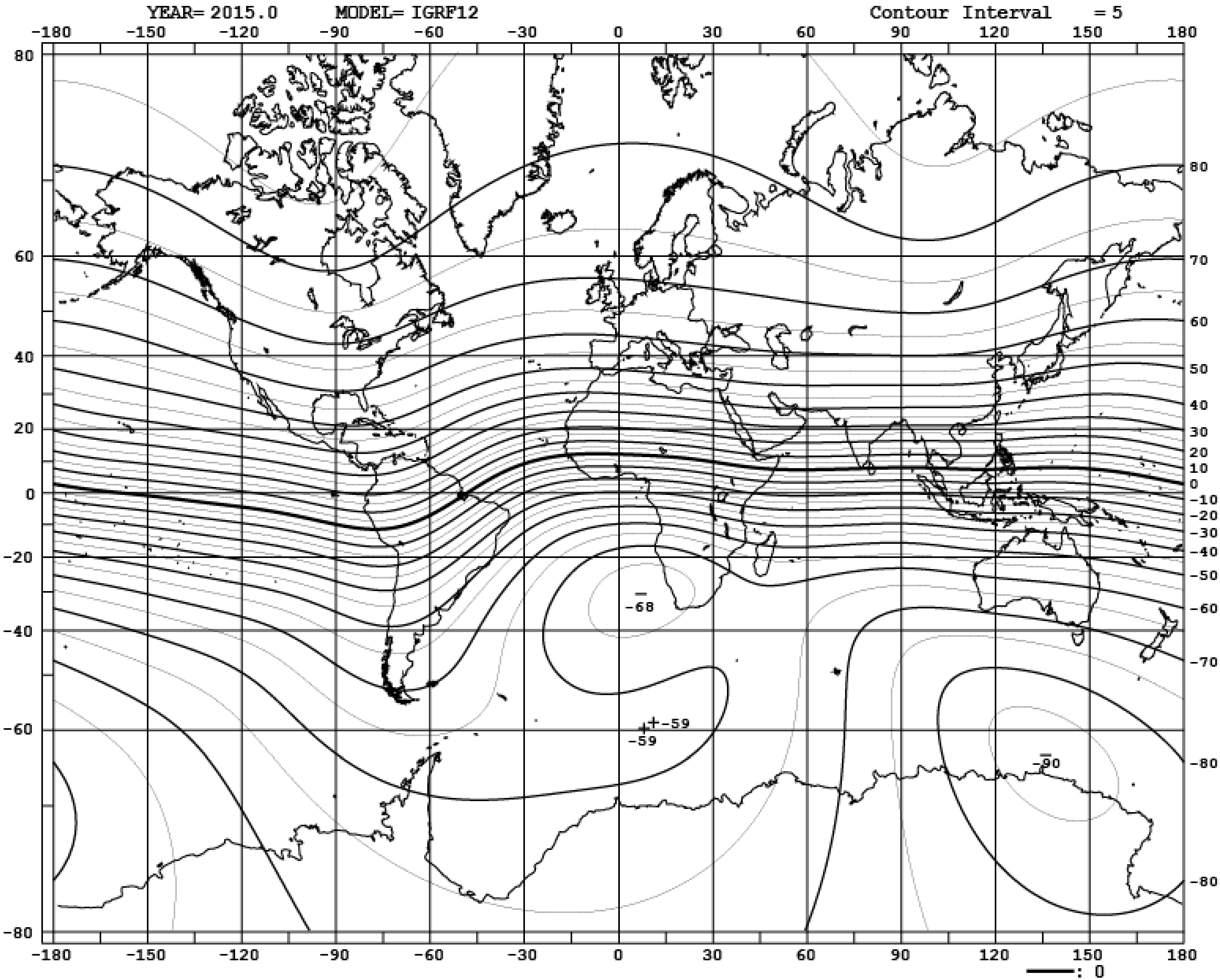
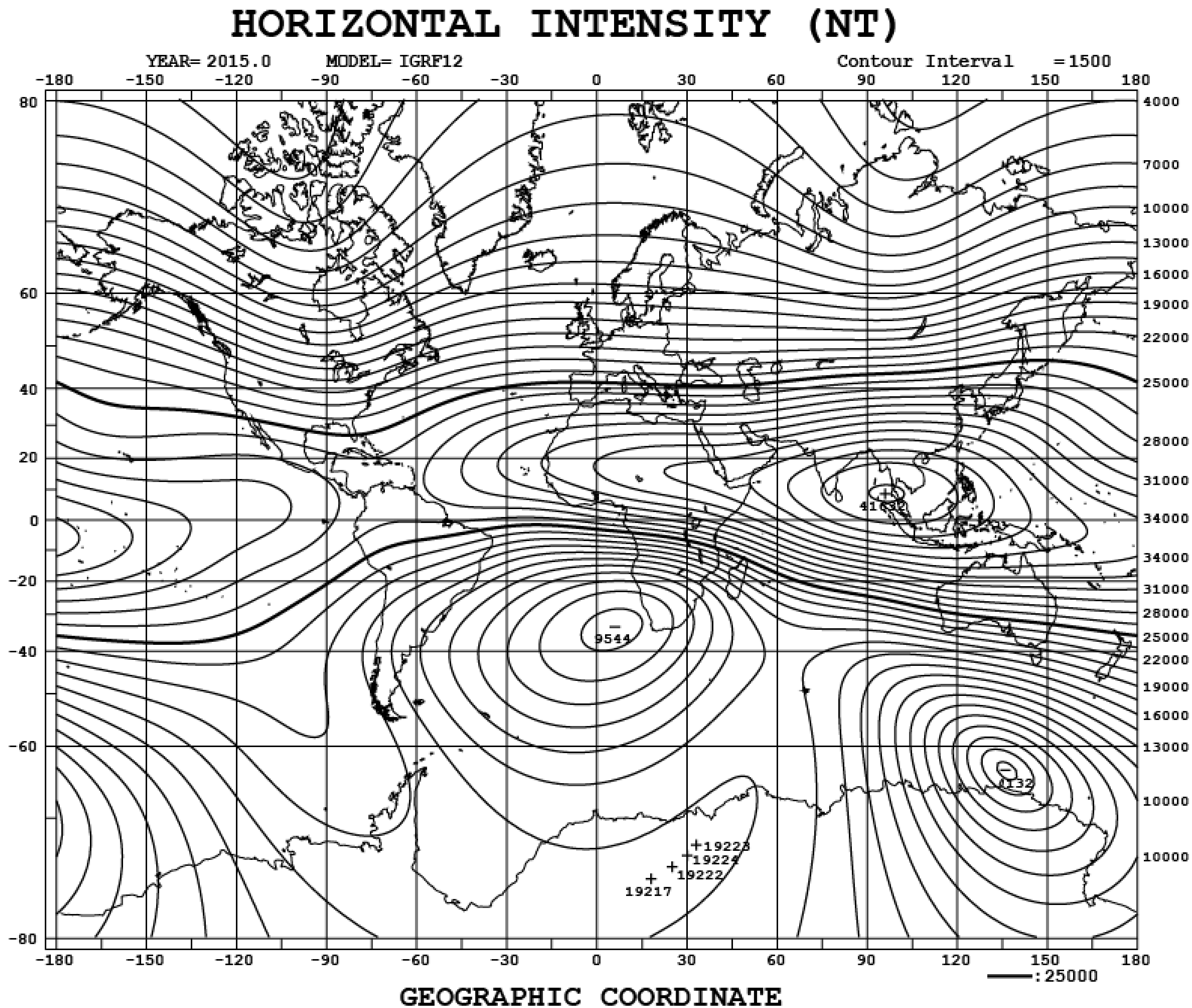
MODEL= IGRF12
Contour Interval =5
GEOGRAPHIC COORDINATE
: 0

INCLINATION (DEGREES)



GEOGRAPHIC COORDINATE



HORIZONTAL INTENSITY (NT)
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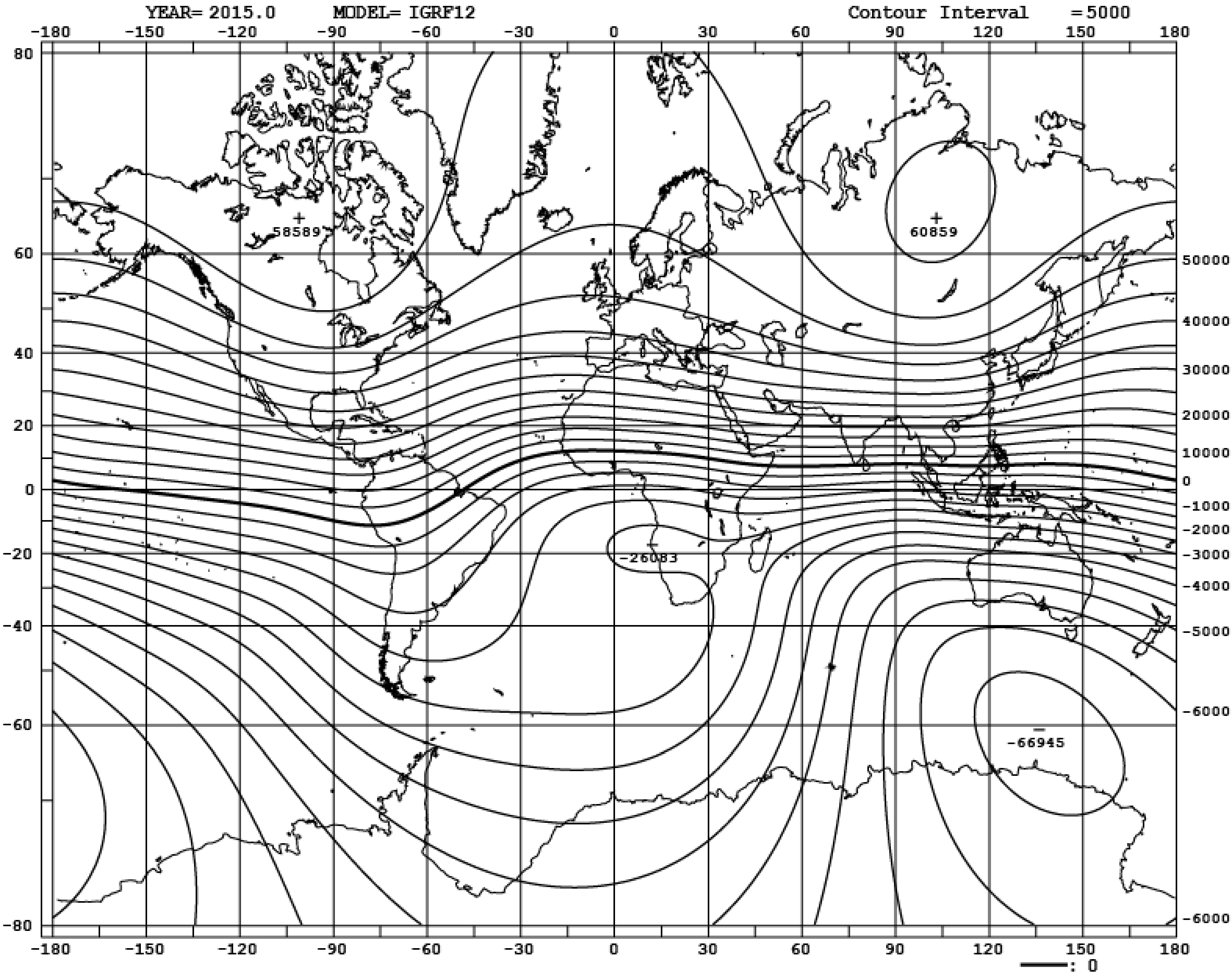
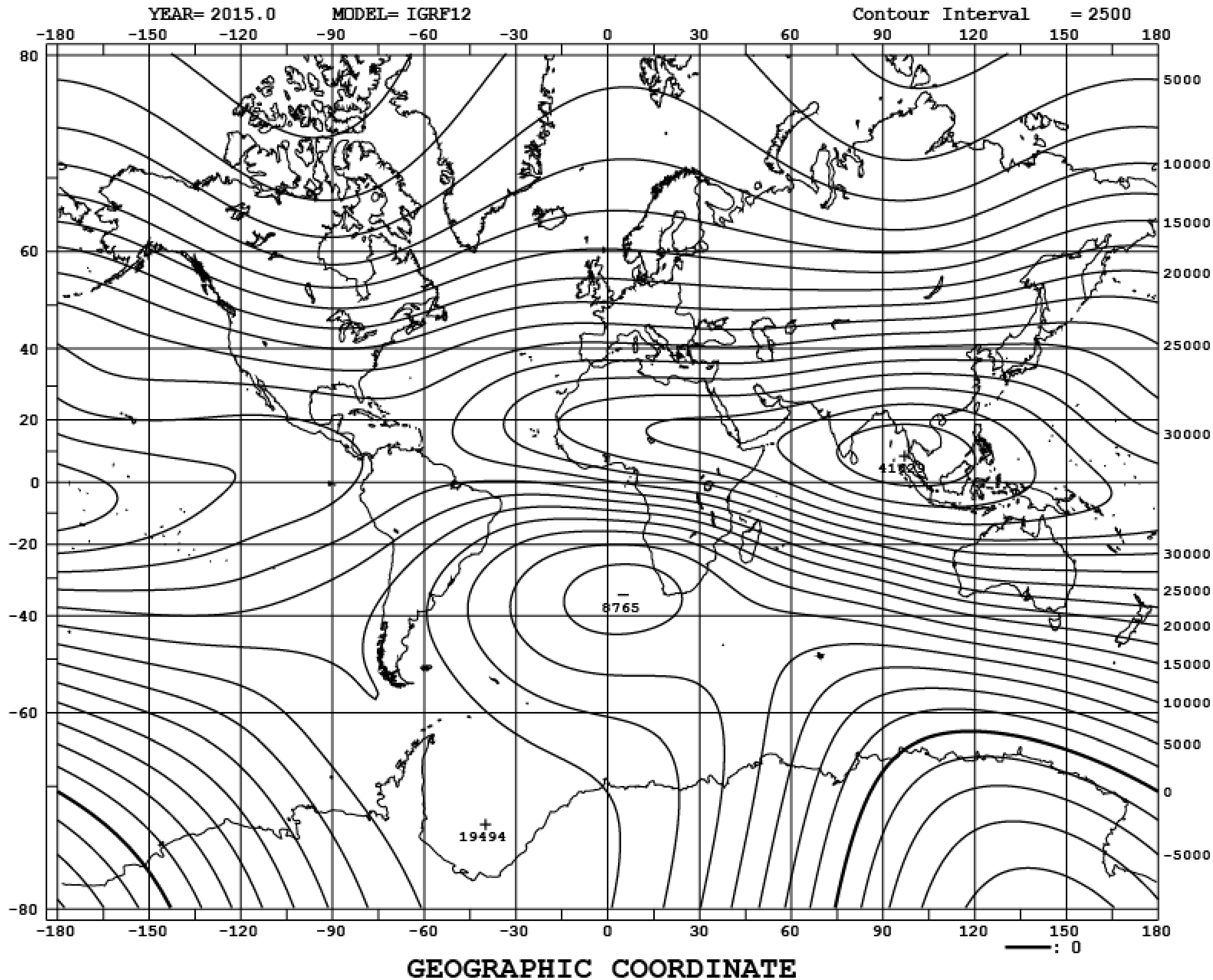
MODEL= IGRF12
Contour Interval =1500
-180 -150 -120 -90 -60 -30 0 30 60 90 120 150 180
80 60 40 20 0 -20 -40 -60 -80
4000
7000
10000
13000
16000
19000
22000
25000
28000
31000
34000
34000
31000
28000
25000
22000
19000
16000
13000
10000
10000
41632
9544
4132
19223
19224
19222
19217
:25000
GEOGRAPHIC COORDINATE

VERTICAL COMPONENT:Z (nT)

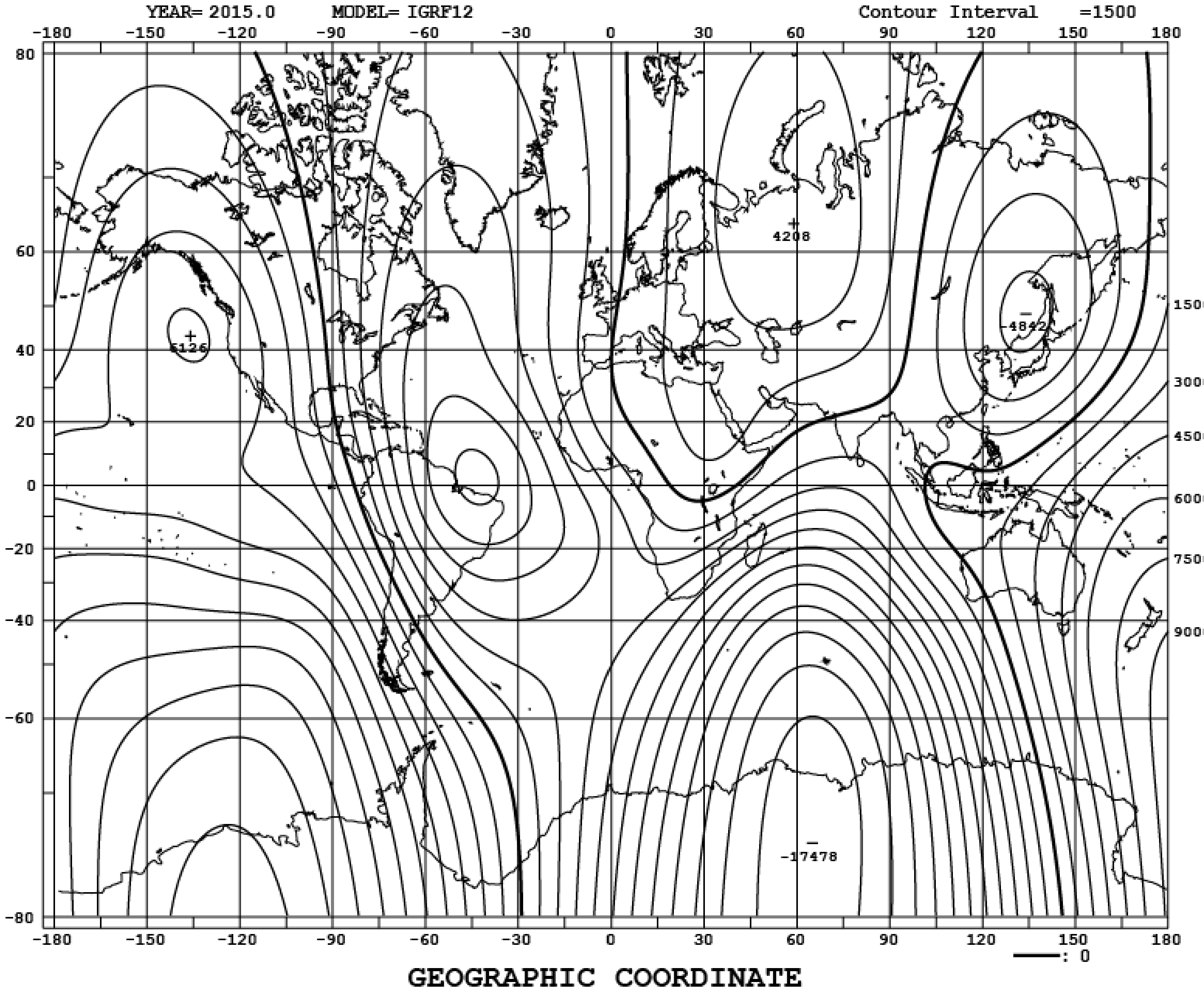


GEOGRAPHIC COORDINATE

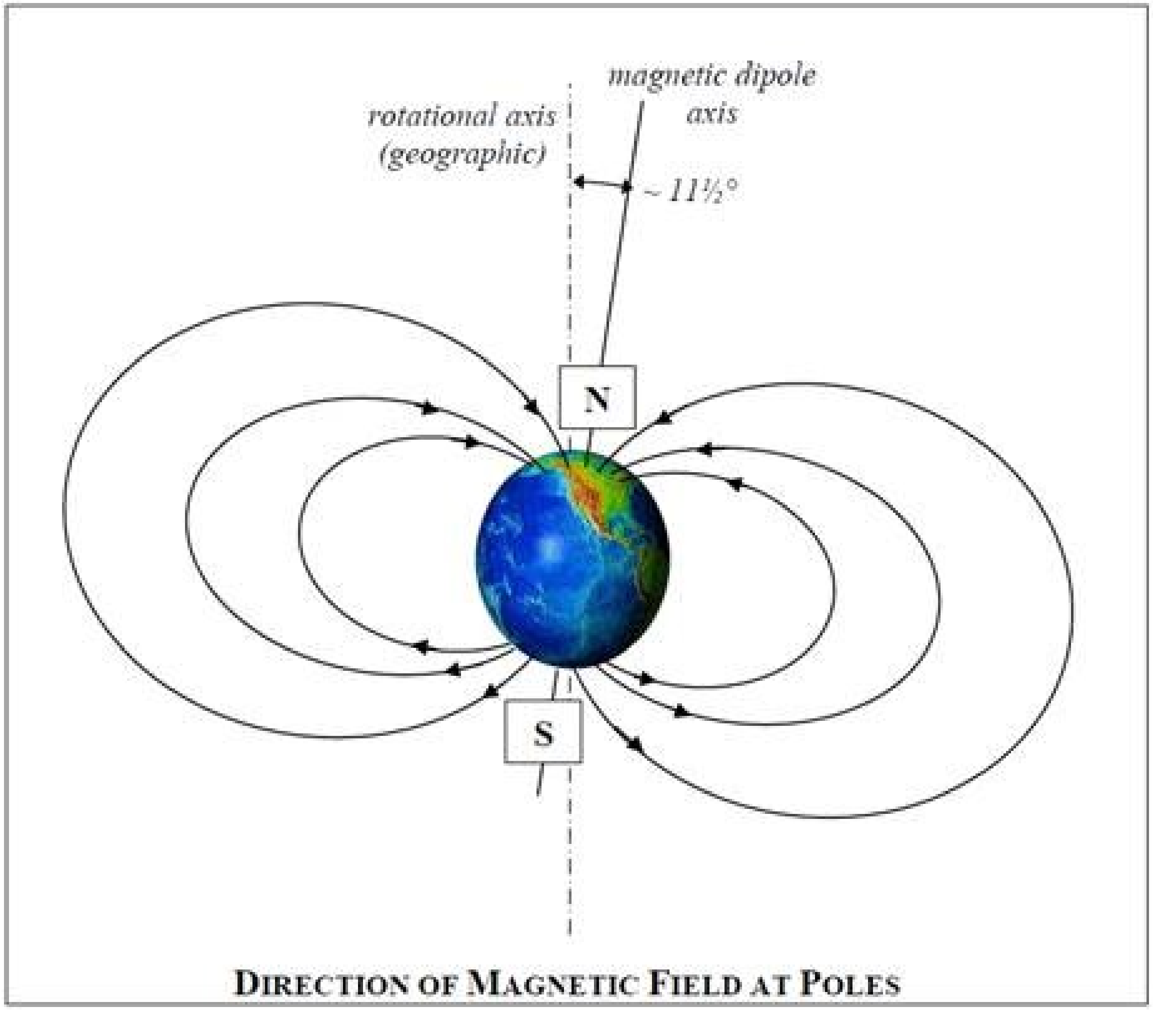
NORTH-SOUTH COMPONENT:X (nT)



EAST-WEST COMPONENT:Y (nT)



Dipole model of the Earth's magnetic field



The dipole model is a first order approximation of the rather complex true magnetic field of the Earth.

For a dipole field apply:

$$\boldsymbol{B}=\frac{1}{r^3}\left[3\left(\boldsymbol{k}_0\cdot\hat{\boldsymbol{e}}_r\right)\hat{\boldsymbol{e}}_r-\boldsymbol{k}_0\right]$$

$$B_r=\frac{-2k_0}{r^3}\sin\lambda$$

$$B_\lambda=\frac{k_0}{r^3}\cos\lambda$$

$$B_\varphi=0$$

$$B=\frac{k_0}{r^3}\sqrt{1+3\sin^2\lambda}$$

$k_0=\mu_0 m/4\pi$ $[\mathrm{Tm}^3=\mathrm{Nm}^2/\mathrm{A}]$

m $[\mathrm{Am}^2]$: dipole moment (strength of the dipole)

$\mu_0=4\pi 10^{-7}\ \mathrm{N/A}^2$: vacuum permeability

For the Earth's dipole:

$k_0=8\times10^{15}\ \mathrm{Tm}^3$

At the magnetic poles $\lambda=\pm90^o$:

$$\boldsymbol{B}=\frac{\mp2k_0}{r^3}\hat{\boldsymbol{e}}_r$$

and B has the maximum value:

$$B=\frac{2k_0}{R_E^3}\approx60\,\mu\mathrm{T}\text{ for the Earth}$$

$\left(\text{Earth radius } 1\ R_E=6371.2\ \mathrm{km}\right)$

At the magnetic equator $\lambda=0^o$:

$$\boldsymbol{B}=\frac{k_0}{r^3}\hat{\boldsymbol{e}}_\lambda$$

and B has the minimum value:

$$B_0=\frac{k_0}{R_E^3}\approx30\,\mu\mathrm{T}\text{ for the Earth}$$

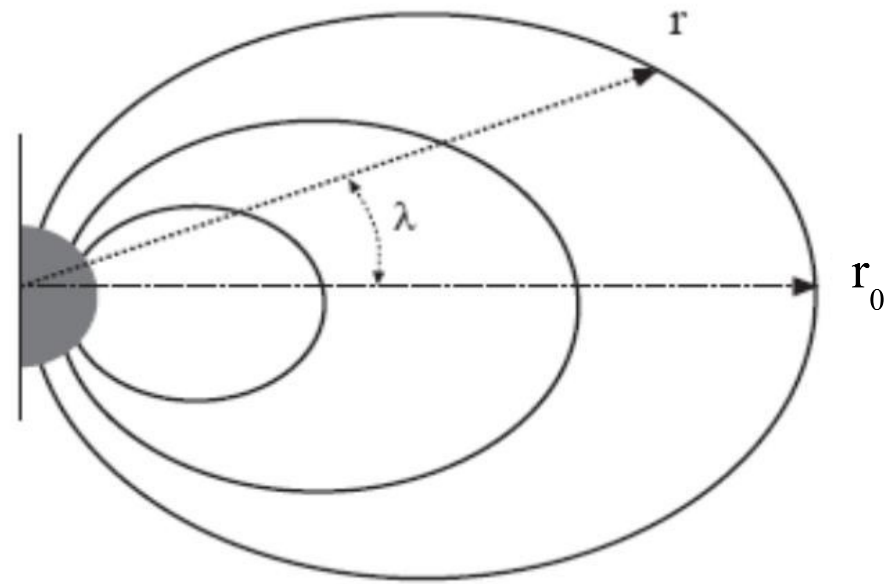
Using B_0, the dipole equations can be written as:

$$B_r=-2B_0\sin\lambda$$

$$B_\lambda=B_0\cos\lambda$$

$$B=B_0\sqrt{1+3\sin^2\lambda}$$

Equation of a dipole field line



Line elements: $d\boldsymbol{r} = dr\,\hat{\boldsymbol{e}}_r + r\,d\theta\,\hat{\boldsymbol{e}}_\theta + r\sin\theta\,d\varphi\,\hat{\boldsymbol{e}}_\varphi$

Equation for a dipole magnetic field line:

$$\frac{dr}{B_r} = \frac{rd\lambda}{B_\lambda}$$

$\rightarrow r = r_0 \cos^2\lambda$,

where r_0 is the radius of the field line at the equator ($\lambda = 0$).

Two parameters, the radius r_0 and the (constant) longitude φ_0, determine each field line.

The so-called L parameter often used in near-Earth space physics is defined as:

$L = r_0 / R_E$,

where R_E is the radius of the Earth.

For a given L value, the field line reaches the Earth's surface at the latitude:

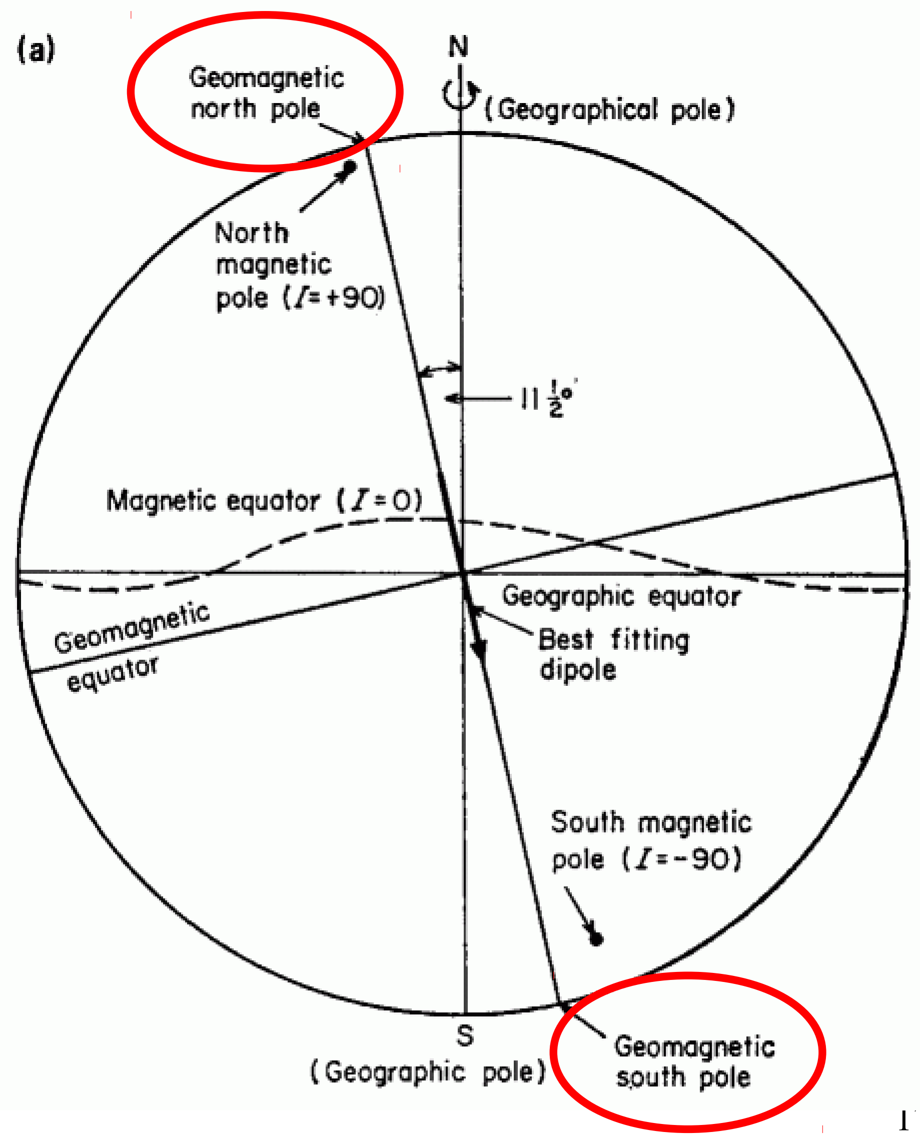
$$\lambda_E = \arccos\frac{1}{\sqrt{L}}$$

Different dipole models of the Earth's magnetic field

1. Axial dipole
   - Dipole located at the center of the Earth.
   - Dipole oriented along the Earth's rotational axis (dipole poles and geographic poles at the same locations).
2. Tilted dipole
   - Dipole located at the center of the Earth.
   - Dipole tilted ~11^{o} from the rotational axis (dipole poles and geographic poles at different locations).
   - In 2009, the northern dipole pole was at 80^{o}N 72^{o}W and moving ~5 km/year northeast.
3. Displaced dipole
   - Tilted dipole (2.) that has been displaced ~500 km from the center of the Earth toward the northern geographic pole (magnetic center of the Earth).
   - Magnetic field not symmetric in the northern and southern hemisphere.

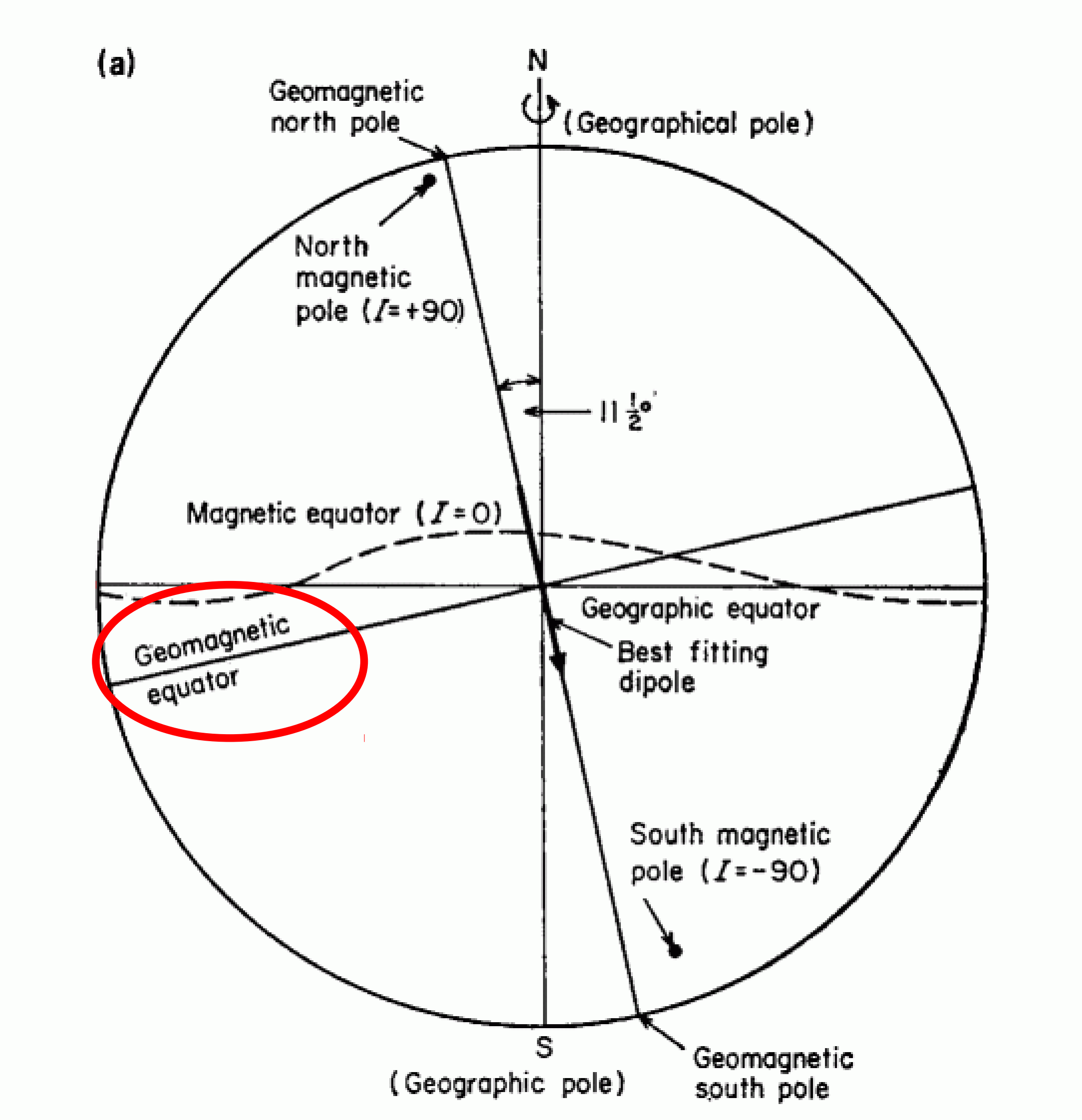
Geomagnetic poles

- Based on global models of the geomagnetic field.
- Models of this type, such as the International Geomagnetic Reference Field (IGRF) include an equivalent (but fictional) magnetic dipole at the center of the Earth in their representation of the field.
- The dipole defines an axis that intersects the Earth's surface at two antipodal points. These points are called geomagnetic poles.
- The axis of the equivalent dipole is currently inclined about 10° to the Earth's rotation axis.
  - North geomagnetic pole in the northern hemisphere (S-pole of a magnet).
  - South geomagnetic pole in the southern hemisphere (N-pole of a magnet).
- These model dip poles do not agree with the measured dip pole positions. The geomagnetic poles cannot be located by direct local measurement.
- The locations of the geomagnetic poles change with time.



Geomagnetic equator

- For a dipole field, the equator is a great circle (the intersection of a sphere and a plane which passes through the center point of the sphere) which is tilted with respect to the geographic equator by the same angle by which the dipole axis is tilted from the rotational axis.

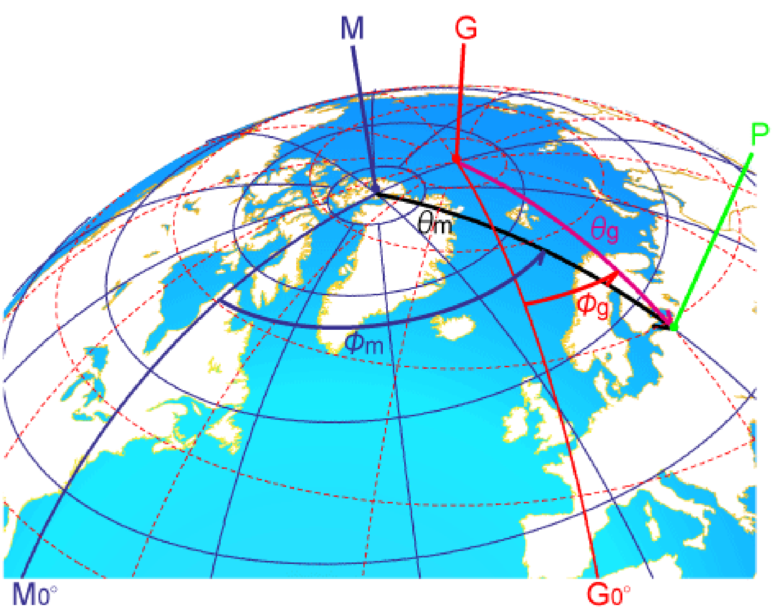


Virtual geomagnetic poles

- Locations of poles calculated from locally measured H and Z by assuming a dipole field.
- Each location may have different virtual poles. Global averages of the virtual poles coincide with the geomagnetic poles.

Geomagnetic coordinates

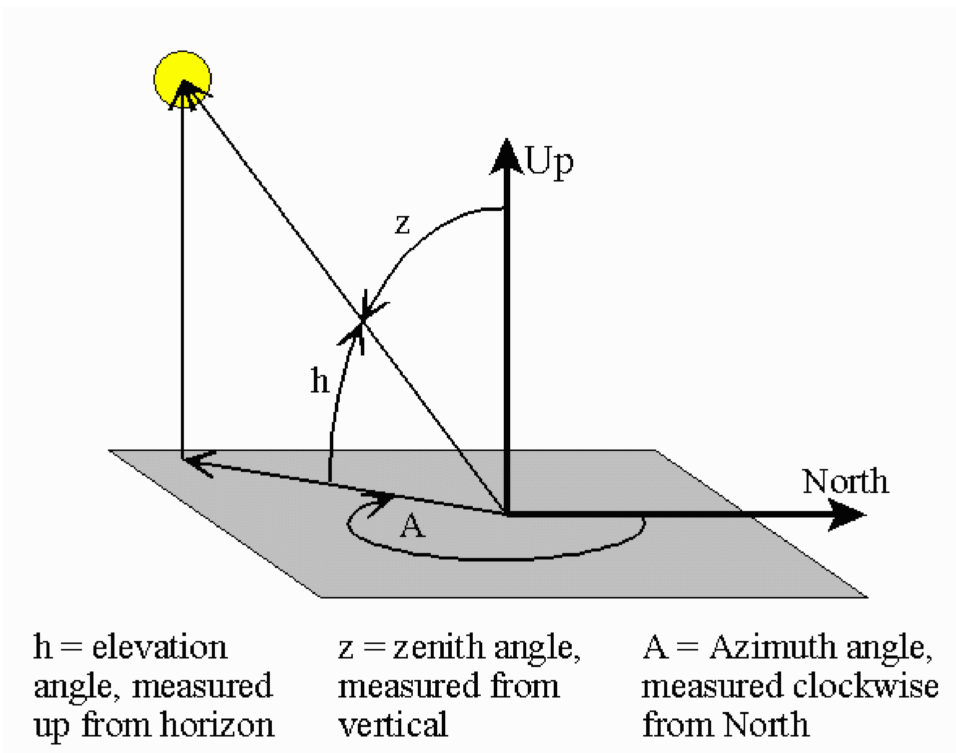
- For the study of many phenomena connected with the Earth's magnetic field, the use of dipole or geomagnetic coordinates is more practical than the use of geographic ones.
- The geomagnetic zero-meridian is the meridian running from the geographic north pole through the geomagnetic north pole to the geographic south pole.
- Normally, the accuracy of 0.1^0 is sufficient for the geomagnetic coordinates.

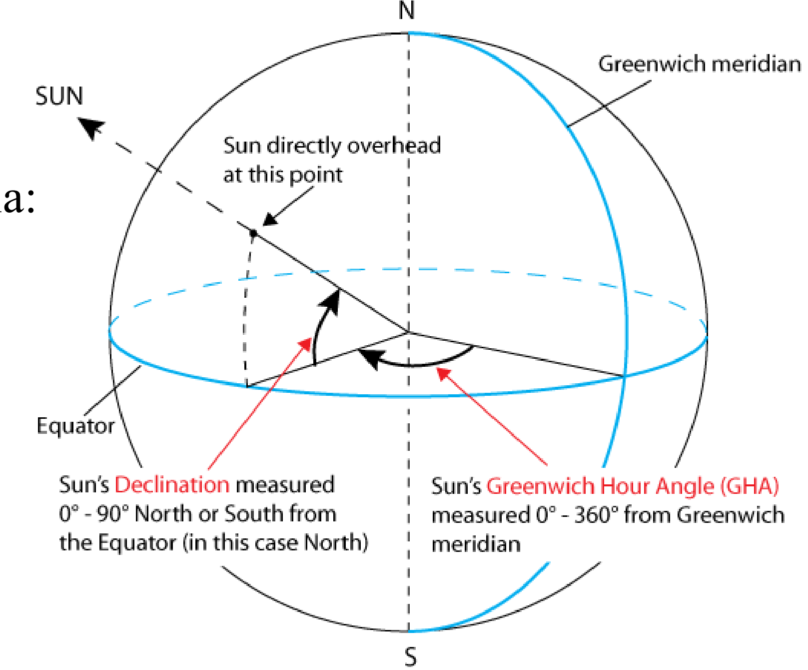


G : Geographic pole M : Geomagnetic pole P : A point
θg: Geocentric colatitude (=90−Geocentric latitude)
Φg: Geographic longitude
θm: Geomagnetic colatitude (=90−Geomagnetic latitude)
Φm: Geomagnetic longitude

Geomagnetic time

- Geomagnetic time is another unit used in connection with studies of geomagnetic disturbances of solar origin.
- It is defined as the angle between the geomagnetic meridian through the station and the one through the Sun. It can therefore be found by computing the geomagnetic longitude of the station and the azimuth of the Sun. At stations in middle and low latitudes, the geomagnetic time differs only slightly from the geographic local time.





$T' = t' + 12$ h apparent geomagnetic local time

t' geomagnetic hour angle can be calculated from the formula:

$$\cot(t' - \varphi') = \pm \frac{\sin\lambda_0 \cos(t - \varphi + \varphi_0) - \cos\lambda \tan\delta}{\sin(t - \varphi + \varphi_0)}$$

λ', φ' geomagnetic latitude and longitude

λ, φ geographic latitude and longitude

T local mean time

$t = T + t_e - 12$ h hour angle of the apparent Sun

δ declination of the Sun

$\lambda_{0,}\, \varphi_0$ geographic latitude and longitude of the north geomagnetic pole

These formulas give an accuracy of $\pm 0.14^o$ for t_e and 0.2^o for δ:

$t_e = 0.00429718^o + 0.107029^o \cos\beta - 1.83788^o \sin\beta - 0.837378^o \cos 2\beta - 2.34048^o \sin 2\beta$

$\delta = 0.39637^o - 22.913^o \cos\beta - 4.0254^o \sin\beta - 0.38720^o \cos 2\beta + 0.05197^o \sin 2\beta$

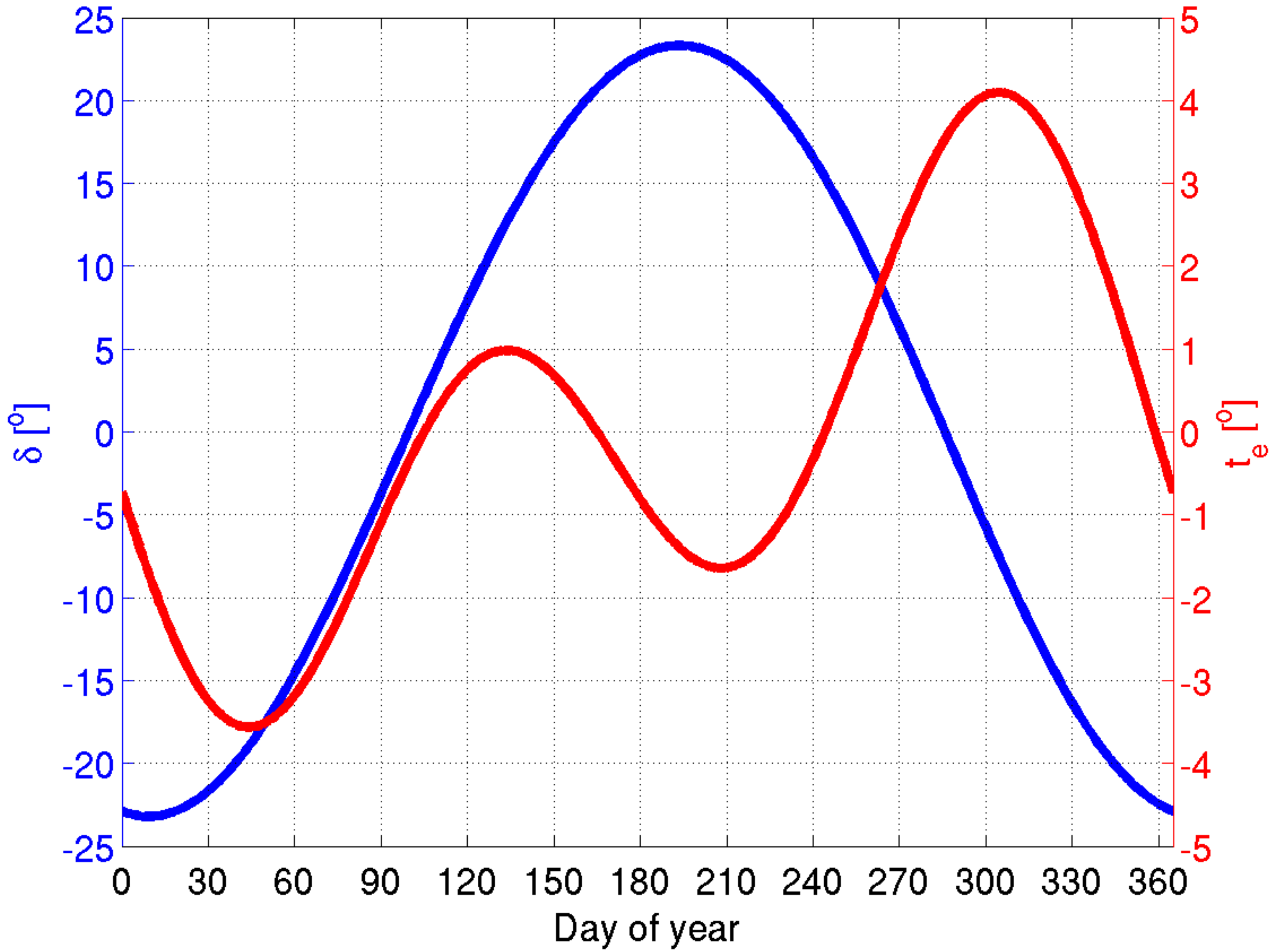
$\beta = 360^o \cdot day/365$ (day$=0$ is Jan 1)

t_e varies roughly between $\pm 4^{o.}$

δ is the angle between the Earth's equatorial plane and the direction of the Sun.

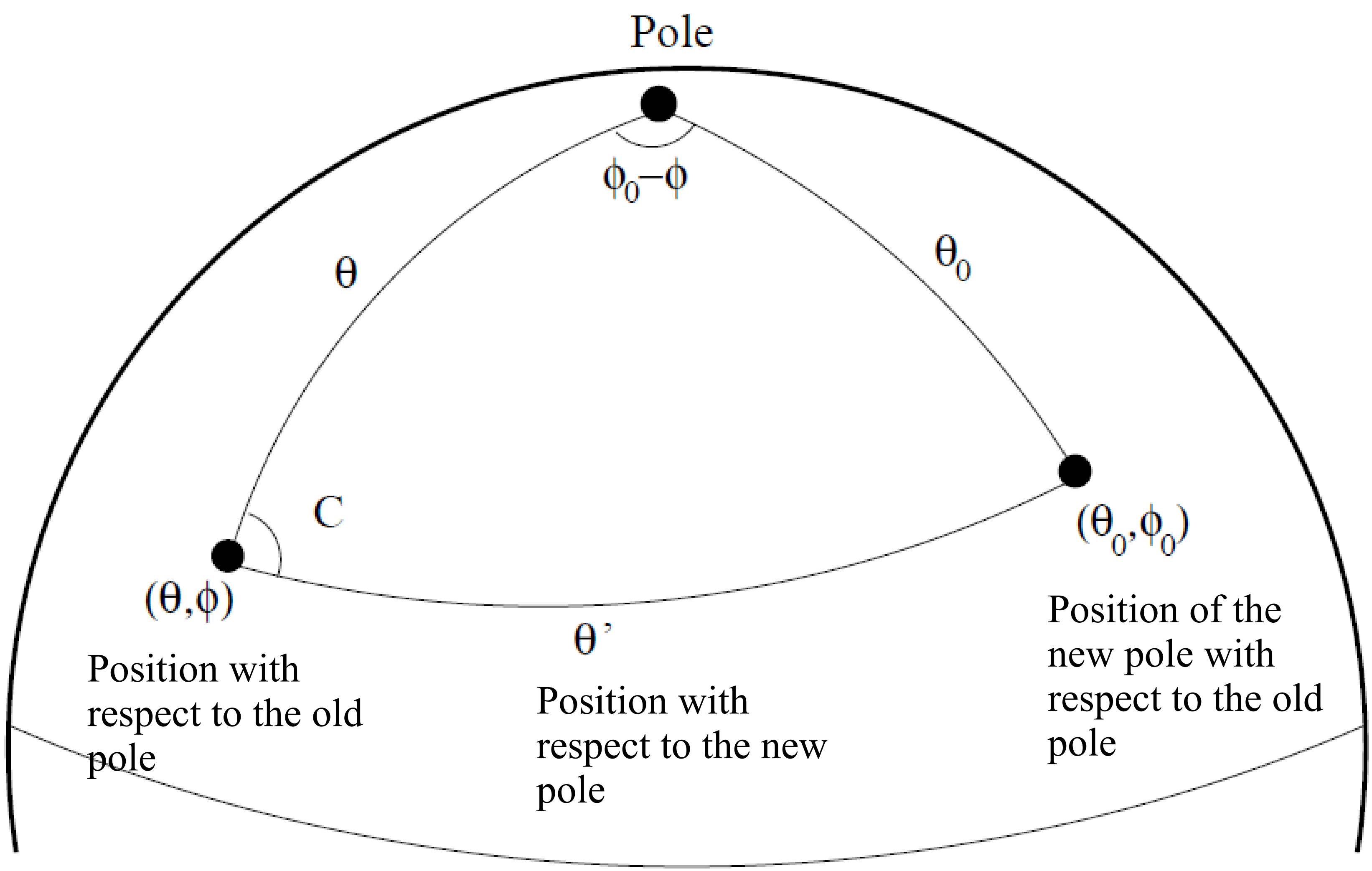
It varies from $+23.5^o$ (summer solstice, northern midsummer) to -23.5^o (winter solstice).

$t_e = t + 12\ \mathrm{h} - T$ is called the equation of time.



- The equation of time describes the discrepancy between two kinds of solar time. These are
  - apparent solar time, which directly tracks the motion of the sun
  - mean solar time, which tracks a fictitious "mean" sun with noons 24 hours apart.
- Apparent (or true) solar time can be obtained by measurement of the current position (hour angle) of the Sun.
  - The solar hour angle is an expression of time, expressed in angular measurement, usually degrees, from solar noon. At solar noon the hour angle is 0 degrees, with the time before solar noon expressed as negative degrees, and the local time after solar noon expressed as positive degrees.
- Mean solar time, for the same place, would be the time indicated by a steady clock set so that over the year its differences from apparent solar time average to zero.

Rotation of the frame of reference



Position vector with respect to the old pole:
$$\boldsymbol{r} = x\hat{\boldsymbol{e}}_x + y\hat{\boldsymbol{e}}_y + z\hat{\boldsymbol{e}}_z$$
Position vector of the new pole with respect to the old pole:
$$\boldsymbol{r}_0 = x_0\hat{\boldsymbol{e}}_x + y_0\hat{\boldsymbol{e}}_y + z_0\hat{\boldsymbol{e}}_z$$
Position vector with respect to the new pole:
$$\boldsymbol{r}' = x'\hat{\boldsymbol{e}}_{x'} + y'\hat{\boldsymbol{e}}_{y'} + z'\hat{\boldsymbol{e}}_{z'}$$

Magnetic field vector with respect to the old pole:
$$\boldsymbol{B} = B_x\hat{\boldsymbol{e}}_x + B_y\hat{\boldsymbol{e}}_y + B_z\hat{\boldsymbol{e}}_z$$
Magnetic field vector with respect to the new pole:
$$\boldsymbol{B}' = B_{x'}\hat{\boldsymbol{e}}_{x'} + B_{y'}\hat{\boldsymbol{e}}_{y'} + B_{z'}\hat{\boldsymbol{e}}_{z'}$$

Transformations:
$$\boldsymbol{r}' = R\cdot\boldsymbol{r}$$
$$\boldsymbol{r} = R^{-1}\cdot\boldsymbol{r}'$$
$$\boldsymbol{B}' = R\cdot\boldsymbol{B}$$
$$\boldsymbol{B} = R^{-1}\boldsymbol{B}'$$

Rotation by angle α with respect to the x axis:

$$Rx(\alpha)=\begin{pmatrix}1 & 0 & 0\\ 0 & \cos\alpha & \sin\alpha\\ 0 & -\sin\alpha & \cos\alpha\end{pmatrix}$$

$$R=\begin{pmatrix}R_{11} & R_{12} & R_{13}\\ R_{21} & R_{22} & R_{23}\\ R_{31} & R_{32} & R_{33}\end{pmatrix}$$

Rotation by angle β with respect to the y axis:

$$Ry(\beta)=\begin{pmatrix}\cos\beta & 0 & -\sin\beta\\ 0 & 1 & 0\\ \sin\beta & 0 & \cos\beta\end{pmatrix}$$

$$R^{-1}=\begin{pmatrix}R_{11} & R_{21} & R_{31}\\ R_{12} & R_{22} & R_{32}\\ R_{13} & R_{23} & R_{33}\end{pmatrix}$$

Rotation by angle γ with respect to the z axis:

$$Rz(\gamma)=\begin{pmatrix}\cos\gamma & \sin\gamma & 0\\ -\sin\gamma & \cos\gamma & 0\\ 0 & 0 & 1\end{pmatrix}$$

Example 1: From geographic to dipole coordinates $R=Ry(\theta_0(t))\cdot Rz(\varphi_0(t))$

Example 2: From dipole to geographic coordinates $R=(Ry(\theta_0(t))\cdot Rz(\varphi_0(t)))^{-1}$

For the north dipole pole, $\theta_0(2010.0)\approx 90^o-79.9^o$ and $\varphi_0(2010.0)\approx 288.0^o$

Note that the dipole coordinates depend on time

➔ the corresponding epoch must always be provided.