Chapter 4
Multipole model of the Earth's magnetic field
Previously

A measurement of the geomagnetic field at any given point and time consists of a superposition of fields from different sources:

- Internal sources:
  - Core or main field: A hydrodynamic dynamo in the Earth's fluid outer core (2900 – 5100 km depth) produces over 99% of the Earth's magnetic field.
  - Crustal or anomalous or lithospheric field: The magnetic field caused by magnetized rocks in the lithosphere (< 50 km depth) can locally exceed the strength of the Earth's main field, but globally constitutes <1% of the field.

- External sources:
  - Solar activity drives electric currents in the Earth's ionosphere and magnetosphere (> 100 km altitude) which cause irregular magnetic field variations with periods from seconds to hours.

The first order approximation of the Earth's rather complex magnetic field is the dipole model.
Content

- Spherical harmonic expansion of the Earth's magnetic field
- Properties of the spherical harmonic expansion
- International Geomagnetic Reference Field (IGRF)
Multipole model of the Earth's magnetic field

- Generalization of the dipole model of the Earth's magnetic field.
- Basic assumption: the Earth's magnetic field can be represented as a superposition of the fields created by several multipole magnets located at the center of the Earth.
- The simplest multipole magnet is the dipole, then quadrupole (four poles), octupole (eight poles), etc.
Spherical harmonics

- Spherical harmonics are a series of special functions defined on the surface of a sphere.

- As Fourier series are a series of functions used to represent functions on a circle, spherical harmonics are a series of functions that are used to represent functions defined on the surface of a sphere.

- Spherical harmonics are defined as the angular portion of a set of solutions to Laplace's equation in three dimensions.

- Spherical harmonics are functions defined in terms of spherical coordinates and organized by wavelength.

\[
Y_{l}^{m}(\theta, \phi) = \sqrt{\frac{2l+1}{4\pi} \frac{(l-m)!}{(l+m)!}} P_{l}^{m}(\cos \theta) e^{im\phi}
\]

- \( \theta \) co-latitude
- \( \phi \) longitude
- \( l \) degree of spherical harmonic
- \( m \) order of spherical harmonic
- \( P_{l}^{m}(\cos \theta) \) associated Legendre function of the first kind
The first few Legendre functions $P_n(x)$ are:

$P_0(x) = 1$

$P_1(x) = x$

$P_2(x) = \frac{1}{2}(3x^2 - 1)$

$P_3(x) = \frac{1}{2}(5x^3 - 3x)$
Unnormalized associated Legendre functions of the first kind

The unnormalized associated Legendre functions $P_n^m(x)$ are related to the Legendre functions $P_n(x)$ by:

$m = 0$: $P_n^0(x) = P_n(x)$

$m \neq 0$: $P_n^m(x) = (-1)^m (1 - x^2)^{m/2} \frac{d^m}{dx^m} P_n(x)$

Matlab function: $P = \text{legendre}(n,x)$
The first few unnormalized associated Legendre functions $P_n^m(x)$ are:

$P_0^0(x) = 1$
$P_1^0(x) = x$
$P_1^1(x) = -(1 - x^2)^{1/2}$
$P_2^0(x) = \frac{1}{2} (3x^2 - 1)$
$P_2^1(x) = -3x(1 - x^2)^{1/2}$
$P_2^2(x) = 3(1 - x^2)$
$P_3^0(x) = \frac{1}{2} (5x^3 - 3x)$
$P_3^1(x) = -\frac{3}{2} (5x^2 - 1)(1 - x^2)^{1/2}$
$P_3^2(x) = 15x(1 - x^2)$
$P_3^3(x) = -15(1 - x^2)^{3/2}$
Visual representations of the first few real spherical harmonics. Blue portions represent regions where the function is positive, and yellow portions represent where it is negative. The distance of the surface from the origin indicates the value of $Y_l^m(\theta, \phi)$ in angular direction $(\theta, \phi)$. 
Wavelength related to harmonic degree

- Spherical harmonics have
  - $n-m$ zeros on parallels in $\pi$ radians of co-latitude
  - $m$ zeros on meridians in $\pi$ radians of longitude.
- Spherical harmonics with $m = 0$ are called zonal, i.e., the functions are independent of the longitude $\phi$.
- Spherical harmonics with $m = l$ are called sectorial, i.e., they represent bands of longitude.
- Spherical harmonics with $m \neq l \neq 0$ are called tesseral.
Although the spherical harmonics are functions on a two-dimensional surface, it is sometimes convenient to characterize them by a one-dimensional “wavelength” $\lambda$. Since a spherical harmonic has $n$ zeros on $\pi$ radians, $\lambda$ is taken to be:

$$\lambda = \frac{2\pi R_E \sin \theta}{n}$$

$\lambda = 2\pi R_E \sin \theta$ for $n = 6$
\[ \nabla \times B = 0 \rightarrow B = -\nabla U, \]
where \( U \) is the scalar potential.

\[ \nabla \cdot B = 0 \rightarrow \nabla^2 U = 0 \text{ (Laplace equation)} \]

\[ \nabla^2 U = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial U}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial U}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2}{\partial \varphi^2} = 0 \]

There are two types of solutions:
- potential \( U_i \) due to sources internal to the Earth \((r < R_E)\)
- potential \( U_e \) due to sources external to the Earth \((r > R_E)\)
such that \( U = U_i + U_e \).
The solutions are given as multipole or spherical harmonic expansions:

\[ U_i(r, \theta, \varphi, t) = R_E \sum_{n=1}^{\infty} \left( \frac{R_E}{r} \right)^{n+1} \sum_{m=0}^{n} \left( g_n^m(t) \cos m\varphi + h_n^m(t) \sin m\varphi \right) SP_n^m(\cos \theta) \]

\[ U_e(r, \theta, \varphi, t) = R_E \sum_{n=1}^{\infty} \left( \frac{R_E}{r} \right)^{-n} \sum_{m=0}^{n} \left( q_n^m(t) \cos m\varphi + s_n^m(t) \sin m\varphi \right) SP_n^m(\cos \theta) \]

\( r \) radius
\( R_E \) Earth radius (\( R_E = 6371.2 \) km)
\( \theta \) co-latitude
\( \varphi \) longitude
\( t \) time
\( n \) degree of multipole (a multipole of degree \( n \) has \( 2^n \) poles)
\( m \) order of multipole
\( g, h, q, s \) spherical harmonic coefficients (describe the strength of the multipole magnet in nT)
\( SP_n^m(\cos \theta) \) Schmidt semi-normalized associated Legendre function of the first kind
Schmidt semi-normalized associated Legendre functions

Schmidt semi-normalized associated Legendre functions \( SP^m_n(x) \) are related to the unnormalized associated Legendre functions \( P^m_n(x) \) by:

\[
\begin{align*}
&m=0: \quad SP^0_n(x) = P^0_n(x) = P_n(x) \\
&m \neq 0: \quad SP^m_n(x) = (-1)^m \sqrt{\frac{2(n-m)!}{(n+m)!}} P^m_n(x)
\end{align*}
\]

Matlab function: \( P = \text{legendre}(n,x,'sch') \)
The first few Schmidt semi-normalized associated Legendre functions $SP^m_n(x)$ are:

$SP^0_0(x) = 1$

$SP^0_1(x) = x$

$SP^1_1(x) = (1 - x^2)^{1/2}$

$SP^0_2(x) = \frac{1}{2} (3x^2 - 1)$

$SP^1_2(x) = \sqrt{3} x (1 - x^2)^{1/2}$

$SP^2_2(x) = \frac{\sqrt{3}}{2} (1 - x^2)$

$SP^0_3(x) = \frac{1}{2} (5x^3 - 3x)$

$SP^1_3(x) = \frac{\sqrt{6}}{4} (5x^2 - 1)(1 - x^2)^{1/2}$

$SP^2_3(x) = \frac{\sqrt{15}}{2} x (1 - x^2)$

$SP^3_3(x) = \frac{\sqrt{10}}{4} (1 - x^2)^{3/2}$
Spherical harmonic expansion of the magnetic field

\[ B = - \nabla U = - \left( \frac{\partial U}{\partial r} \hat{e}_r + \frac{1}{r} \frac{\partial U}{\partial \theta} \hat{e}_\theta + \frac{1}{r \sin \theta} \frac{\partial U}{\partial \varphi} \hat{e}_\varphi \right) \]

Sources internal to the Earth:

\[ B_{ri}(r, \theta, \varphi, t) = - \frac{\partial U_i}{\partial r} = \sum_{n=1}^{n_{\text{max}}} \left( n + 1 \right) \left( \frac{R_E}{r} \right)^{n+2} \sum_{m=0}^{n} \left( g_n^m(t) \cos m \varphi + h_n^m(t) \sin m \varphi \right) \frac{SP_n^m(\cos \theta)}{\sin \theta} \]

\[ B_{\theta i}(r, \theta, \varphi, t) = - \frac{1}{r} \frac{\partial U_i}{\partial \theta} = \sum_{n=1}^{n_{\text{max}}} \left( \frac{R_E}{r} \right)^{n+2} \sum_{m=0}^{n} \left( g_n^m(t) \cos m \varphi + h_n^m(t) \sin m \varphi \right) \frac{dSP_n^m(\cos \theta)}{d\theta} \]

\[ B_{\varphi i}(r, \theta, \varphi, t) = - \frac{1}{r \sin \theta} \frac{\partial U_i}{\partial \varphi} = \sum_{n=1}^{n_{\text{max}}} \left( \frac{R_E}{r} \right)^{n+2} \sum_{m=0}^{n} \left( -g_n^m(t) \sin m \varphi + h_n^m(t) \cos m \varphi \right) \frac{mSP_n^m(\cos \theta)}{\sin \theta} \]

Sources external to the Earth:

\[ B_{re}(r, \theta, \varphi, t) = - \frac{\partial U_e}{\partial r} = \sum_{n=1}^{n_{\text{max}}} n \left( \frac{R_E}{r} \right)^{-n+1} \sum_{m=0}^{n} \left( q_n^m(t) \cos m \varphi + s_n^m(t) \sin m \varphi \right) SP_n^m(\cos \theta) \]

\[ B_{\theta e}(r, \theta, \varphi, t) = - \frac{1}{r} \frac{\partial U_e}{\partial \theta} = \sum_{n=1}^{n_{\text{max}}} \left( \frac{R_E}{r} \right)^{-n+1} \sum_{m=0}^{n} \left( q_n^m(t) \cos m \varphi + s_n^m(t) \sin m \varphi \right) \frac{dSP_n^m(\cos \theta)}{d\theta} \]

\[ B_{\varphi e}(r, \theta, \varphi, t) = - \frac{1}{r \sin \theta} \frac{\partial U_e}{\partial \varphi} = \sum_{n=1}^{n_{\text{max}}} \left( \frac{R_E}{r} \right)^{-n+1} \sum_{m=0}^{n} \left( -q_n^m(t) \sin m \varphi + s_n^m(t) \cos m \varphi \right) \frac{mSP_n^m(\cos \theta)}{\sin \theta} \]
In practice, the sum to infinity has to be truncated at $n = n_{\text{max}}$, determined by the number of available observations.

There are $n_{\text{max}}^2 + 2n_{\text{max}}$ coefficients in the expansion.

In practice, the number of observations is generally much larger than this.

The values of the coefficients are determined using the least squares method.
\[
\frac{d}{d \theta} S P^m_n (\cos \theta) \text{ from recurrence formulas}
\]

\[m > 0:\]
\[
(1 - x^2) \frac{d}{dx} P_n^m(x) = (n+m)(n-m+1)\sqrt{1-x^2} P_n^{m-1}(x) + m x P_n^m(x)
\]

\[
\frac{d}{dx} S P_n^m(x) = (-1)^m \sqrt{\frac{2(n-m)!}{(n+m)!}} \frac{d}{dx} P_n^m(x)
\]

\[
\frac{d}{d \theta} P_n^m (\cos \theta) = \frac{d \cos \theta}{d \theta} \frac{d}{d (\cos \theta)} P_n^m(\cos \theta) = -\sin \theta \frac{d}{d (\cos \theta)} P_n^m(\cos \theta)
\]

\[m = 0:\]
\[
(1 - x^2) \frac{d}{dx} P_n(x) = n P_{n-1}(x) - n x P_n(x)
\]

\[
\frac{d}{dx} S P_n(x) = \frac{d}{dx} P_n(x)
\]

\[
\frac{d}{d \theta} P_n (\cos \theta) = \frac{d \cos \theta}{d \theta} \frac{d}{d (\cos \theta)} P_n(\cos \theta) = -\sin \theta \frac{d}{d (\cos \theta)} P_n(\cos \theta)
\]
Properties of the spherical harmonic expansion
In case of the internal field:

- Large wavelengths ($n < 14$, approximately) are associated with the main field:
  - $n = 1$: dipole component
  - $2 \leq n < 14$: anomalous or non-dipole components

  (Note: The terms $2 \leq n < 14$ are considered anomalous relative to the dipole field whereas the terms $n > 14$ representing the crustal field are considered anomalous relative to the main field. Thus, the anomalous component is always relative, and the reference level should be mentioned.)

- Smaller wavelengths ($n > 14$) are associated with the magnetic anomalies of the crust.
平均磁力场

在磁力场由于内部源的情况下，平均磁力场的平方

\( B_n^m \) 平方平均值在地球表面 \( S \) 定义为：

\[
\langle (B_n^m)^2 \rangle = \frac{1}{4\pi} \oint (B_n^m)^2 dS
\]

可以证明，对于每个多项级数 (\( n \)) 的结果为：

\[
\langle (B_n^2) \rangle = (n + 1) \sum_{m=0}^{n} (g_n^m)^2 + (h_n^m)^2
\]

公式可以用来估计不同多项级数的相对强度：
(模型: POMME-6.2 2005.0)

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\[
\log_{10}(\langle(B_n)^2\rangle) = -0.57 \cdot n + 9.3
\]

- **Main field (n<14)**
- **Surface anomalies (n>14)**
Minimum wavelength at the equator ($\theta = 90^\circ$) associated with the spherical harmonic of degree $n$. 
If \( r = R_C < R_E \) (source region of the magnetic field at the surface of the liquid core):

\[
\langle (B_n)^2 \rangle_C = \left( \frac{R_E}{R_C} \right)^{2(n+2)} \langle (B_n)^2 \rangle_E
\]

\[
\rightarrow \log_{10} \left( \langle (B_n)^2 \rangle_E \right) = -2(n+2) \log_{10} \left( \frac{R_E}{R_C} \right) + \log_{10} \left( \langle (B_n)^2 \rangle_C \right)
\]

\[
\rightarrow k = -\log_{10} \left( \frac{R_E}{R_C} \right)^2 \approx -0.57 \text{ (components } n<14 \text{ in the figure)}
\]

(Assume that \( \log_{10} \left( \langle (B_n)^2 \rangle_C \right) \) does not depend on \( n \).)

\[
\rightarrow R_C = 10^{-0.57} R_E \approx 0.52 R_E \approx 3300 \text{ km}
\]

When \( R_C \rightarrow R_E, k \rightarrow 0 \) (components \( n>14 \) in the figure)
International Geomagnetic Reference Field (IGRF)
http://www.ngdc.noaa.gov/IAGA/vmod/igrf.html
• Geomagnetic field models are represented as spherical harmonic expansions of a scalar magnetic potential. Such a model can then be evaluated at any desired location to provide the magnetic field vector.

• IGRF was introduced by the International Association of Geomagnetism and Aeronomy (IAGA) in 1968 in response to the demand for a standard spherical harmonic representation of the Earth's main field.

• IGRF can be considered to consist of two parts:
  – mathematical functions that describe how each multipole field changes as a function of latitude, longitude, and radius (“geometry of the multipole field”)
  – coefficients \((2n+1)\) for each \(n \geq 1\) associated with each multipole (“strength of the multipole field”)
• IGRF is meant to give a reasonable approximation, near and above the Earth's surface, to that part of the Earth's magnetic field which has its origin inside the surface.

• The model is updated at 5-year intervals. The latest (as of May 2015) is IGRF-12.

• At any one epoch, the IGRF specifies the numerical coefficients of a truncated spherical harmonic series.
  – For dates until 2000 the truncation is at \( n = 10 \), with 120 coefficients.
  – From 2000 the truncation is at \( n = 13 \), with 195 coefficients.

• Such a model is specified every 5 years, for epochs 1900.0, 1905.0, etc. For dates between the model epochs, coefficient values are given by linear interpolation.

• For the 5 years after the most recent epoch there is a linear secular variation model for forward extrapolation; this SV model is truncated at \( n = 8 \), so has 80 coefficients (in effect the next 40 or 115 coefficients are defined to be zero).

\[
g_n^m(t) = g_n^m(t_0) + g_n^{m'}(t_0)(t-t_0) + g_n^{m''}(t_0) \frac{(t-t_0)^2}{2!} + \ldots
\]

\[
h_n^m(t) = h_n^m(t_0) + h_n^{m'}(t_0)(t-t_0) + h_n^{m''}(t_0) \frac{(t-t_0)^2}{2!} + \ldots
\]
“Health warning”

- When using IGRF, to avoid ambiguity you should state explicitly which IGRF generation you are using.
- Because of the time variation of the field, really good models can only be produced for times when there is global coverage by satellites measuring the vector field. This occurred in:
  - 1979 – 1980: Magsat
  - 1999 – : Ørsted, CHAMP, SWARM
- At some time later, IGRF models are replaced by definitive DGRF models (“definitive = we will not be able to do significantly better in the future”).
- Interpolate between the appropriate DGRF models if they exist. If there is not a DGRF model, then use the appropriate IGRF model.
- If you measure the magnetic field at a point on the Earth's surface, do not expect to get the value predicted by the IGRF:
  - The numerical coefficients will not be correct: the model field produced will differ from the actual field.
  - Because of truncation, the IGRF model represents only the lower spatial frequencies (longer wavelengths) of the field: higher spatial frequency components are not accounted for.
  - There are also other contributions to the observed field (both natural and man-made) the IGRF is not trying to model: buildings, parked cars, magnetization of crustal rocks, traffic, DC electric trains and trams, electric currents in the ionosphere and magnetosphere, etc.
IGRF-12

- Full name: IGRF 12th generation
- Short name: IGRF-12
- Valid for: 1900.0 – 2020.0
- Definitive for: 1945.0 – 2010.0
- Reference: Thébault et al., Earth Planets and Space, 2015
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Magnetic maps of the Earth

from
http://www.geomag.bgs.ac.uk/education/earthmag.html
Map of declination (degrees east or west of true north) at 2015.0

Isogonic line ($D = \text{constant}$)

Agonic line ($D = 0^\circ$)
Map of predicted annual rate of change of declination (degrees/year east or west) for 2015.0-2020.0.
Map of inclination (angle in degrees up or down that magnetic field vector is from the horizontal) at 2015.0

Magnetic equator ($I = 0^\circ$)
Map of predicted annual rate of change of inclination (degrees/year up or down) for 2015.0-2020.0
Minima around magnetic poles

Map of horizontal intensity at 2015.0

Maximum around equator
Map of predicted annual rate of change of horizontal intensity for 2015.0-2020.0
Map of vertical intensity at 2015.0
Map of predicted annual rate change of vertical intensity for 2015.0-2020.0
Map of total intensity at 2015.0

Maximum associated with north magnetic pole (note asymmetry between hemispheres)

Maximum associated with south magnetic pole
Map of predicted annual rate of change of total intensity for 2015.0-2020.0
Average multipole strengths

Average annual rate of change of multipole strengths
Maps of the dipole and non-dipole components of IGRF in Finland
Altitude-Adjusted Corrected Geomagnetic Coordinates (AACGM)

Examples of determining AACGM coordinates for four geographic locations along the prime meridian. Red lines represent IGRF field lines emanating from geographic starting locations at 50°, 40°, 30° latitude, and ending at the Earth-centered magnetic dipole equator. AACGM coordinates are given by the coordinates of the dipole field lines, shown in green. The magenta line shows the IGRF field line starting at 20° latitude, which intersects the surface of Earth before the dipole equator. AACGM coordinates are undefined for such locations. The region near the magnetic dip equator (orange line) which includes these field lines is marked by yellow lines on Earth’s surface. From: Shepherd (2014).