Alfven Waves and the Ionosphere

Heikki Vanhamäki

Finnish Meteorological Institute,
Space Research Unit

Research seminar on Sun-Earth Connections, spring 2006

Outline

- Background
- Properties of Alfven waves
  - Dispersion relation from ideal MHD
  - Different wave modes and their properties
- Alfven waves and the ionosphere
  - Ionospheric reflection coefficient
  - Field line oscillations
  - Ionospheric Alfven resonator

I: Background

There is a huge number of different wave modes in plasma, but **Alfven waves** are arguably the most important.

They carry energy, momentum, angular momentum and currents between different regions in the magnetosphere-ionosphere system.

Alfven speed v_A is the speed at which information is transmitted in plasma
==> Limit for time-step / grid spacing in MHD simulations.
(Courant stability condition)

Ionospheric disturbances create Alfven waves,
magnetospheric Alfven waves are reflected and modified at the ionosphere
==> ionosphere has an active role in M-I coupling

ILMATIETEEN LAITOS
METEOROLOGISKA INSTITUTET
FINNISH METEOROLOGICAL INSTITUTE

II: Properties of Alfven waves

Derivation (see e.g. Koskinen, 2001)

Ideal MHD equations:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{V}) = 0$$ Continuity of mass

$$\rho \frac{\partial \boldsymbol{V}}{\partial t} + \rho (\boldsymbol{V} \cdot \nabla) \boldsymbol{V} = -\nabla p + \boldsymbol{J} \times \boldsymbol{B}$$ Momentum equation

$$\nabla p = v_s^2 \nabla \rho$$ Equation of state (v_s is speed of sound)

$$\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{J}$$

$$\nabla \times \boldsymbol{E} = \frac{-\partial \boldsymbol{B}}{\partial t}$$

$$\boldsymbol{E} + \boldsymbol{V} \times \boldsymbol{B} = 0$$ Ohm's law in ideal MHD

Eliminate pressure, current and electric field.

We end up with

$$\frac{\partial \rho}{\partial t}+\nabla\cdot(\rho \boldsymbol{V})=0$$

$$\rho\frac{\partial \boldsymbol{V}}{\partial t}+\rho(\boldsymbol{V}\cdot\nabla)\boldsymbol{V}=-v_s^2\nabla\rho+\frac{1}{\mu_0}(\nabla\times\boldsymbol{B})\times\boldsymbol{B}$$

$$\frac{\partial \boldsymbol{B}}{\partial t}=\nabla\times(\boldsymbol{V}\times\boldsymbol{B})$$ Convection equation

Now assume static situation with *uniform magnetic field* $\boldsymbol{B_0}$ (in z-direction), *zero velocity* and *uniform density* ρ_0. Then introduce *small* perturbations.

$$\boldsymbol{B}(\boldsymbol{r},t)=\boldsymbol{B_0}+\boldsymbol{B_1}(\boldsymbol{r},t)$$
$$\rho(\boldsymbol{r},t)=\rho_0+\rho_1(\boldsymbol{r},t)$$
$$\boldsymbol{V}(\boldsymbol{r},t)=\boldsymbol{V_1}(\boldsymbol{r},t)$$

Linearize equations by taking only 1st order terms (0th order trivial, 2nd order very small):

$$\frac{\partial \rho_1}{\partial t}+\nabla\cdot(\rho_0 \boldsymbol{V_1})=0$$

$$\rho_0\frac{\partial \boldsymbol{V_1}}{\partial t}=-v_s^2\nabla\rho_1+\frac{1}{\mu_0}(\nabla\times\boldsymbol{B_1})\times\boldsymbol{B_0}$$

$$\frac{\partial \boldsymbol{B_1}}{\partial t}=\nabla\times(\boldsymbol{V_1}\times\boldsymbol{B_0})$$

These give equation for the velocity disturbance:

$$\frac{\partial^2 \boldsymbol{V_1}}{\partial t^2}-v_s^2\nabla(\nabla\cdot\boldsymbol{V_1})+v_A^2\hat{\boldsymbol{e}}_z\times(\nabla\times[\nabla\times(\boldsymbol{V_1}\times\hat{\boldsymbol{e}}_z)])=0$$

where $v_a=\frac{B_0}{\sqrt{\mu_0\rho_0}}$ is Alfven speed and $\boldsymbol{B_0}=B_0\hat{\boldsymbol{e}}_z$

Look for plane wave solutions, $\boldsymbol{V}_{\mathbf{1}}(\boldsymbol{r},t)=\boldsymbol{V}_{\mathbf{1}}e^{i(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)}$

After some algebra the previous equation reads:

$$-\omega^2\boldsymbol{V}_{\mathbf{1}}+(v_s^2+v_A^2)(\boldsymbol{k}\cdot\boldsymbol{V}_{\mathbf{1}})\boldsymbol{k}+v_A^2(\boldsymbol{k}\cdot\hat{\boldsymbol{e}}_{\boldsymbol{z}})[(\boldsymbol{k}\cdot\hat{\boldsymbol{e}}_{\boldsymbol{z}})\boldsymbol{V}_{\mathbf{1}}-(\boldsymbol{V}_{\mathbf{1}}\cdot\hat{\boldsymbol{e}}_{\boldsymbol{z}})\boldsymbol{k}-(\boldsymbol{k}\cdot\boldsymbol{V}_{\mathbf{1}})\hat{\boldsymbol{e}}_{\boldsymbol{z}}]=0$$

This is *dispersion equation* for Alfven waves.

The uniform background magnetic field, $\mathbf{B}_0$, is in z-direction.
Choose

$$\boldsymbol{k}=k(\hat{\boldsymbol{e}}_{\boldsymbol{x}}\sin\theta+\hat{\boldsymbol{e}}_{\boldsymbol{z}}\cos\theta)$$
$$\boldsymbol{V}_{\mathbf{1}}=V_{1x}\hat{\boldsymbol{e}}_{\boldsymbol{x}}+v_{1y}\hat{\boldsymbol{e}}_{\boldsymbol{y}}+V_{1z}\hat{\boldsymbol{e}}_{\boldsymbol{z}}$$

The dispersion equation in component from is

$$\begin{bmatrix} -\omega^2+k^2v_A^2+k^2v_s^2\sin^2\theta & 0 & k^2v_s^2\sin\theta\cos\theta \\ 0 & -\omega^2+k^2v_A^2\cos^2\theta & 0 \\ k^2v_s^2\sin\theta\cos\theta & 0 & -\omega^2+k^2v_s^2\cos^2\theta \end{bmatrix}\cdot\begin{pmatrix} V_{1x} \\ V_{1y} \\ V_{1z} \end{pmatrix}=0$$

There are non-zero solutions for $\boldsymbol{V}_1$ only if the determinant is zero.
Three different cases:

$\omega^2=k^2v_A^2\cos^2\theta$ Alfven mode

$\omega^2=\frac{k^2}{2}\left(v_A^2+v_s^2\pm\sqrt{(v_A^2+v_s^2)^2-4v_A^2v_s^2\cos^2\theta}\right)$ Fast (+) and slow (-) modes

Alfven mode (aka shear or transversal or intermediate Alfven wave)

Alfven mode dispersion relation $-\omega^2+v_A^2 k_\|^2=0$ (with $k_\|=k\cos\theta$) tells us that the velocity disturbance $\boldsymbol{V}_1$ (and also $\boldsymbol{E}_1$ and $\boldsymbol{B}_1$) satisfies

$$\partial_t^2 \boldsymbol{V}_1 - v_A^2 \nabla_\|^2 \boldsymbol{V}_1 = 0$$

In general this has a solution

$$\boldsymbol{V}_1=\boldsymbol{V}_1^{\uparrow\uparrow}(\boldsymbol{r}_\perp, r_\| - v_A t)+\boldsymbol{V}_1^{\uparrow\downarrow}(\boldsymbol{r}_\perp, r_\| + v_A t)$$

We see that the Alfven mode waves propagate only parallel or anti-parallel to the background magnetic field and not at all perpendicular to it.

From dispersion relation we also get
phase speed $\omega/k=v_A|\cos\theta|$ and group velocity $\nabla_{\boldsymbol{k}}\omega=\pm v_A\hat{\boldsymbol{e}}_\|$
(upper sign for parallel propagation)

NOTE: $\|$ and $\perp$ are with respect to the background field $\mathbf{B}_0$

Properties of the wave fields:

- Velocity disturbance is perpendicular to background field and wave vector, $\boldsymbol{V}_1\cdot\boldsymbol{k}=0=\boldsymbol{V}_1\cdot\boldsymbol{B}_0$
- Magnetic disturbance from convection equation, $\boldsymbol{B}_1=\mp(B_0\boldsymbol{V}_1)/v_A$
- No density or pressure disturbance.
- Electric disturbance from Ohm's law, $\boldsymbol{E}_1=-\boldsymbol{V}_1\times\boldsymbol{B}_0$
- The wave carries a field aligned current (FAC) $j_\|=\mp i B_0 k_\perp V_1/(\mu_0 v_A)$
- Poynting flux is $\boldsymbol{S}=\boldsymbol{E}_1\times\boldsymbol{B}_1/\mu_0=\pm(B_0^2 V_1^2)/(\mu_0 v_A)\hat{\boldsymbol{e}}_\|$

Also note:

- Relation between electric and magnetic disturbances $\boldsymbol{B}_1=\mp(\boldsymbol{E}_1\times\hat{\boldsymbol{e}}_\|)/v_A$
- Perpendicular electric field is a potential field, $(\nabla\times\boldsymbol{E}_{1\perp})_\|=0$
- Relation between FAC and perpendicular electric field $j_\|=\pm\Sigma_A\nabla_\perp\cdot\boldsymbol{E}_{1\perp}$ where $\Sigma_A=1/(\mu_0 v_A)$ is Alfven conductance.

Alfven wave with ***k*** parallel to ***B***$_0$.

Magnetic field is not compressed, only undulating ==> no density disturbance (frozen-in condition).

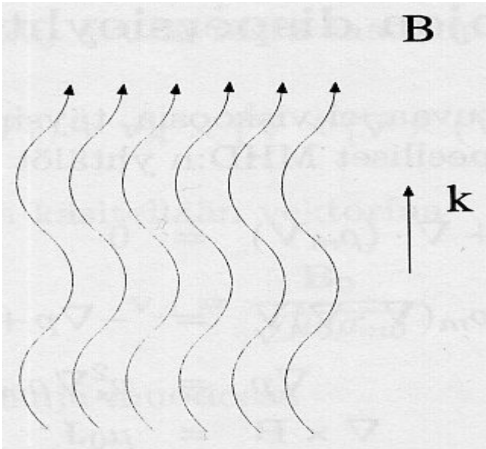


Figure from Koskinen (2001)

ILMATIETEEN LAITOS
METEOROLOGISKA INSTITUTET
FINNISH METEOROLOGICAL INSTITUTE

Fast mode (aka compressional or fast magnetosonic / Alfven wave)

In warm plasma v_s > 0 and dispersion relation is complicated.
Group velocity is not parallel to ***k*** and depends on direction and frequency.

In cold plasma magnetic pressure >> kinetic pressure. Then also v_A >> v_s and the dispersion relation simplifies to $\omega^2 = k^2 v_A^2$
Now group and phase velocities are equal and isotropic, $\nabla_{\boldsymbol{k}} \omega = v_A \hat{\boldsymbol{e}}_{\boldsymbol{k}}$
so the fast mode propagates in all directions.

Properties of the wave fields: (in cold plasma)

- Velocity disturbance is in the direction of the perpendicular wave vector, $\boldsymbol{V_1} \cdot \boldsymbol{B_0} = 0, \quad \boldsymbol{V_1} \cdot \boldsymbol{k} = V_1 k_\perp$
- Magnetic disturbance is $\boldsymbol{B_1} = \left(-B_0 \cos\theta \, \boldsymbol{V_1} + V_1 \sin\theta \, \boldsymbol{B_0} \right) / v_A$

- Electric disturbance is $\boldsymbol{E}_1 = -\boldsymbol{V}_1 \times \boldsymbol{B}_0$
- There is also a density disturbance $\rho_1 = -(\rho_0 V_1 \sin\theta)/v_A$
- Field aligned current is zero, $\boldsymbol{j} \cdot \boldsymbol{B}_0 = 0$
- The Poynting flux is $\boldsymbol{S} = (B_0^2 V_1^2)/(\mu_0 v_A)\hat{\boldsymbol{e}}_k$
- The perpendicular electric field $\boldsymbol{E}_{1\perp}$ is divergence-free,

$$\nabla_\perp \cdot \boldsymbol{E}_{1\perp} = 0, \qquad (\nabla \times \boldsymbol{E}_{1\perp})_{\parallel} = (V_1 B_0 \sin\theta)/v_A$$

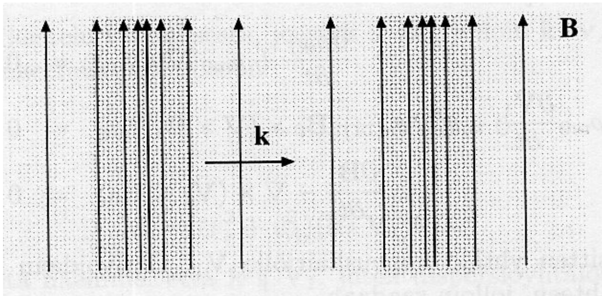


Figure from Koskinen (2001)

ILMATIETEEN LAITOS
METEOROLOGISKA INSTITUTET
FINNISH METEOROLOGICAL INSTITUTE

Slow mode (aka slow magnetosonic or Alfven wave)

This mode disappears in cold plasma.

In the magnetosphere we might have v_A ~ 1000 km/s and v_s ~ 10 km/s.
==> plasma is quite cold.

Home exercise: Derive the properties of slow and fast mode waves in warm plasma.

ILMATIETEEN LAITOS
METEOROLOGISKA INSTITUTET
FINNISH METEOROLOGICAL INSTITUTE

Some complications

• In some situations v_A could exceed c
==> Need to include $c^{-2}\partial_t \boldsymbol{E}$ term.

• In the magnetosphere typical v_A ~1000 km/s, observed geomagnetic pulsation have periods 10 - 600 s ==> wavelength 2-100 Earth radii.
==> Background $\boldsymbol{B}_0$, ρ_0, $\boldsymbol{V}_0$ non-uniform and perhaps time-dependent.

• In the ionosphere the Ohm's law is $\boldsymbol{J} = \overline{\overline{\sigma}} \cdot \boldsymbol{E}$

with
$$\overline{\overline{\sigma}} = \begin{bmatrix} \sigma_P & -\sigma_H & 0 \\ \sigma_H & \sigma_P & 0 \\ 0 & 0 & \sigma_\parallel \end{bmatrix}$$

ILMATIETEEN LAITOS
METEOROLOGISKA INSTITUTET
FINNISH METEOROLOGICAL INSTITUTE

III: Alfven waves and the ionosphere

Reflection at the ionosphere

Simplified model: - $\boldsymbol{B}_0$ is uniform and in z-direction (z positive downward)
- Ionosphere is a thin sheet at z=0 with uniform Hall and Pedersen conductances
- Ideal magnetospheric plasma above, z<0
- Neutral atmosphere below, z>0

Above ionosphere we have the ideal MHD wave modes. Assume that there is no incident fast mode wave, for fast waves are not guided by the magnetic field ==> geometric attenuation.

Electric field of the incident and reflected waves is $\boldsymbol{E}_\perp = \boldsymbol{E}_\perp^\downarrow + \boldsymbol{E}_\perp^\uparrow$

They carry field aligned current $j_\parallel = \Sigma_A \nabla \cdot \left(\boldsymbol{E}_\perp^\downarrow - \boldsymbol{E}_\perp^\uparrow\right)$

In the ionosphere we have Ohm's law $\boldsymbol{J}_\perp = \Sigma_P \boldsymbol{E}_\perp - \Sigma_H \boldsymbol{E}_\perp \times \hat{\boldsymbol{e}}_\parallel$

and the field aligned current is

$$j_\parallel = \nabla \cdot \boldsymbol{J}_\perp = \Sigma_P \nabla \cdot \boldsymbol{E}_\perp + \boldsymbol{E}_\perp \cdot \left(\nabla \Sigma_P\right) - \Sigma_H \hat{\boldsymbol{e}}_\parallel \cdot \left(\nabla \times \boldsymbol{E}_\perp\right) - \left(\boldsymbol{E}_\perp \times \hat{\boldsymbol{e}}_\parallel\right) \cdot \left(\nabla \Sigma_P\right)$$

If we assume *uniform conductances* and *non-rotational electric field*, we have relation

$$\Sigma_P \nabla \cdot \left(\boldsymbol{E}_\perp^\downarrow + \boldsymbol{E}_\perp^\uparrow\right) = \Sigma_A \nabla \cdot \left(\boldsymbol{E}_\perp^\downarrow - \boldsymbol{E}_\perp^\uparrow\right)$$

$$\Rightarrow \boldsymbol{E}_\perp^\uparrow = R_I \boldsymbol{E}_\perp^\downarrow \qquad \text{with } R_I = \frac{\Sigma_A - \Sigma_P}{\Sigma_A + \Sigma_P}$$

Magnetic field above the ionosphere is

$$\boldsymbol{B}^{above} = \frac{1}{v_A}\left(-\boldsymbol{E}_\perp^\downarrow + \boldsymbol{E}_\perp^\uparrow\right) \times \hat{\boldsymbol{e}}_\parallel = \frac{-2\Sigma_P}{v_A\left(\Sigma_A + \Sigma_P\right)} \boldsymbol{E}_\perp^\downarrow \times \hat{\boldsymbol{e}}_\parallel$$

At the ionosphere there is a jump in the magnetic field given by

$$\Delta \boldsymbol{B}_\perp = \mu_0 \boldsymbol{J}_\perp \times \hat{\boldsymbol{e}}_\parallel = \frac{2}{v_A\left(\Sigma_A + \Sigma_P\right)}\left(\Sigma_P \boldsymbol{E}_\perp^\downarrow \times \hat{\boldsymbol{e}}_\parallel - \Sigma_H \boldsymbol{E}_\perp^\downarrow\right)$$

Below the ionosphere the magnetic field is

$$\boldsymbol{B}^{below} = \boldsymbol{B}^{above} + \Delta \boldsymbol{B}_\perp = \frac{-2\Sigma_H}{v_A\left(\Sigma_A + \Sigma_P\right)} \boldsymbol{E}_\perp^\downarrow$$

Magnetic field is rotated by 90 degrees and amplified by Σ_H / Σ_P at the ionospheric boundary.

More realistic model:

- Inhomogenous conductances: Introduce a wave potential for the reflected wave, $E^{\uparrow}_{\perp} = -\nabla\phi$. FAC condition gives a diff. eq. for ϕ (Glassmeier, 1984).

- Inductive processes: Mode conversion occurs in the reflection, incident shear wave --> reflected shear and fast waves (Yoshikawa and Itonaga, 1996; Buchert, 1997).

- Obligue background magnetic field (Sciffer at al., 2004).

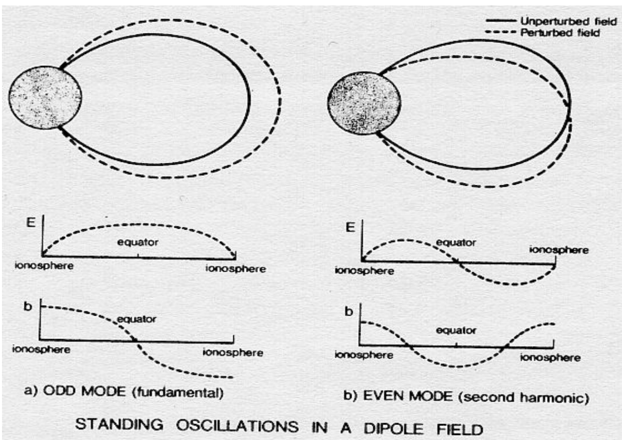
- All of the above + 3D ionosphere + non-linear effects: Numerical, more or less MHD-type approaches (e.g. Streltsov and Lotko, 2004; Lysak, 2004; Dreher, 1997).

Field line oscillations

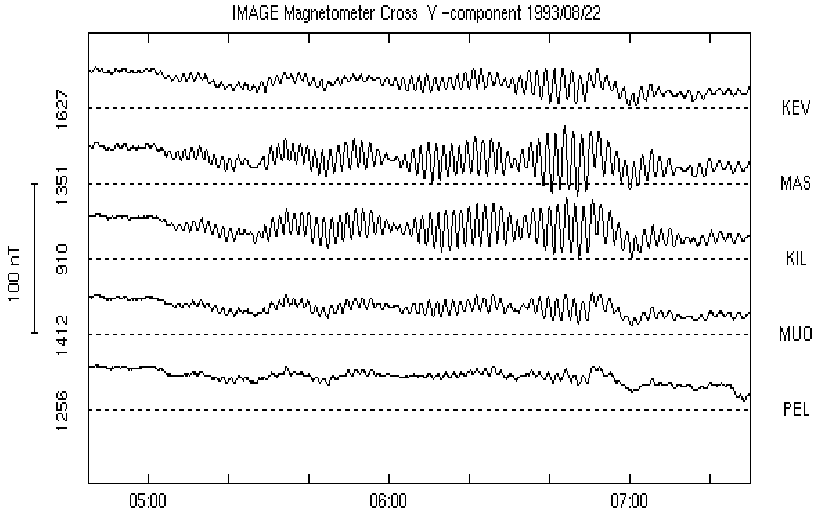
Figure from Hughes (1983)

Standing shear Alfven waves reflecting between northern and southern ionospheres.
==>
Discrete frequency spectra in mHz range.

Analytical solution for Alfven waves in dipole geometry exists.



See e.g. Hughes (1983) or Oulu textbook for further information.



Giant pulsations obserd with the IMAGE magnetometer network.

Ionospheric waveguide

- Under some conditions fast magnetosonic waves can be trapped in the ionospheric F-layer. These waves can propagate across magnetic field
  ==> ionospheric waveguide

- These guided waves are associated with so called pc1 pulsations in the ground magnetic field.

- Fast waves are created locally, when a magnetospheric disturbance is transmitted to ionosphere.
  ==> Waveguide transmits disturbances over large areas in the ionosphere

Results from a simulation where an incident shear wave excites fast mode waves in the waveguide.

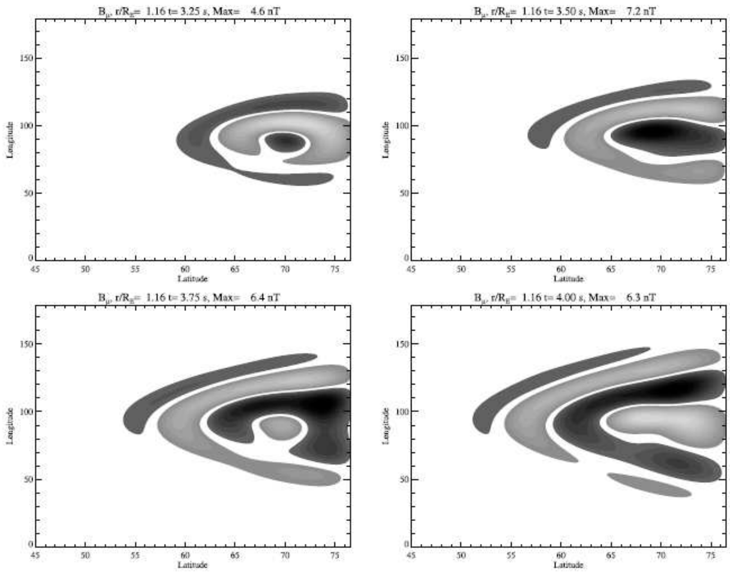


Figure from Lysak (2004)

Ionospheric Afven resonator (IAR)

- Alfven waves are reflected from ionosphere because of the sudden change in conductivity.

- Also large gradients in Alfven velocity cause reflection.

- Alfven velocity has a maximum ~1 Earth radii above the ionosphere.

==> Resonant cavity may form between ionosphere and velocity maximum.

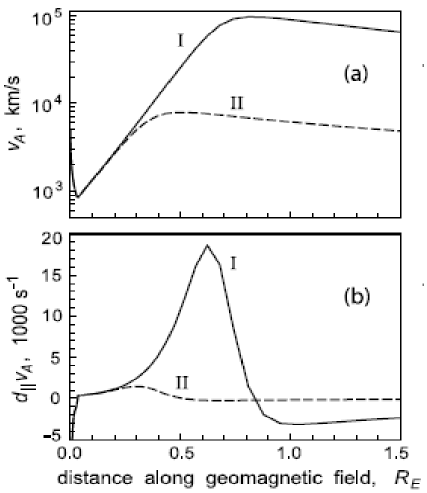


Figure from Streltsov and Lotko (2004)

ILMATIETEEN LAITOS
METEOROLOGISKA INSTITUTET
FINNISH METEOROLOGICAL INSTITUTE

Ionospheric Alfven resonator may amplify weak disturbances

==>

Ionosphere generates small scale structures by itself.

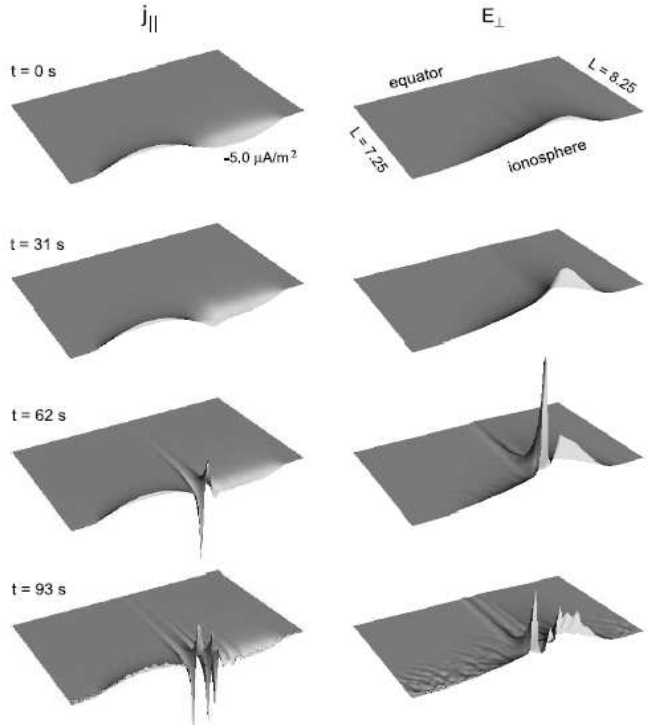


Figure from Streltsov and Lotko (2004)

ILMATIETEEN LAITOS
METEOROLOGISKA INSTITUTET
FINNISH METEOROLOGICAL INSTITUTE

Sources

- Oulu Space Physics Textbook, *http://www.oulu.fi/~spaceweb/textbook/*
- Buchert S., “Magneto-optical Kerr effect for a dissipative plasma”, *J. Plasma Phys.*, **59**, 39-55, 1997.
- Dreher J., “On the self-consistent description of dynamic magnetosphere-ionosphere coupling phenomena with resolved ionosphere”, *J. Geophys. Res*., **102**, 85-94, 1997.
- Glassmeier K.-H., “On the influence of ionospheres with non-uniform conductivity distribution on hydromagnetic waves”, *J. Geophys.*, **54**, 125-137, 1984.
- Hughes W., “Hydromagnetic waves in the magnetosphere”, article in *Solar-terrestrial physics* by R. Carovillano and J. Forbes (eds.), pp 453-477, D. Reidel publishing company, 1983.
- Koskinen H., “Johdatus plasmafysiikkaan ja sen avaruussovellutuksiin”, Limes ry., 2001.
- Lysak R., “Magnetosphere-ionosphere coupling by Alfven waves at midlatitudes”, *J. Geophys. Res.*, **109**, doi:10.1029/2004JA010454, 2004.
- Sciffer M., C. Waters and F. Menk, “Propagation of ULF waves through the ionosphere: Inductive effects for obligue magnetic fileds”, *Ann. Geophys.*, **22**, 1155-1169, 2004.
- Streltsov A. and W. Lotko, “Multiscale electrodynamics of the ionosphere-magnetosphere system”, *J. Geophys. Res.*, **109**, doi:10.1029/2004JA010457, 2004.
- Vogt J., “Alfven wave coupling in the auroral current circuit”, *Surveys in Geophysics*, **23**, 335-377, 2002.
- Yoshikawa A., and M. Itonaga, “Reflection of shear Alfven waves at the ionosphere and the divergent Hall current”, *Geophys. Res. Letters*, **23**, 101-104, 1996.