Electromagnetic Fields Inside the Magnetosphere

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Outline

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  - Magnetic field geometry
  - Magnetospheric convection
  - Magnetospheric current systems
  - Magnetospheric dynamics
- Motivation
- Magnetic field models
  - Historical models
  - Present-day models
- Electric field models
  - Equatorial models
  - Ionospheric models
- Modelling methods
  - $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$
  - $\mathbf{E} \cdot \mathbf{B} = 0$
- Deformation method
  - Motivation
  - Euler potentials
  - Theory
  - Example & Model construction
  - 3D electric fields
- Modelling example
  - SSC, April 06, 2000
  - Dayside compression
Intro to Large-Scale Electromagnetic Fields

Magnetic Field Geometry

- Internal magnetic field
- Dipole tilt
- Solar wind pressure
- Open field lines
- Closed field lines
- Cusp

Intro to Large-Scale Electromagnetic Fields (cont.)

Convection

- Ionosphere:
- Global convection maps to the ionospheric convection
- Typically two-shell pattern (IMF $B_Z < 0$)
- High-latitude convection from noon to midnight
- Return convection at lower latitudes via dawn and dusk
- Can be measured by radars

http://superdarn.jhuapl.edu/
Intro to Large-Scale Electromagnetic Fields (cont.)

Convection (cont.)

- Equatorial plane:
- Earthward convection in the mid-tail
- Plasma flow around the co-rotating plasmasphere to the dayside
- Return convection along the flanks
- Electric field associated with the convection $\sim 0.5\text{mV/m}$
- Cannot be globally measured

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Intro to Large-Scale Electromagnetic Fields (cont.)

Convection (cont.)

- Tail cross-section:
- Convection across the tail lobes towards the plasma sheet
- Turns to earthward convection at the plasma sheet
Intro to Large-Scale Electromagnetic Fields (cont.)

Current Systems

- Cross-tail current
- Ring current
- Magnetopause current
- Field-aligned currents
Intro to Large-Scale Electromagnetic Fields (cont.)

Magnetospheric Dynamics

- Magnetic storms
- Magnetopause compression
- Ring current enhancement
- Substorms
- Tail stretching
- Dipolarization

Why Analytical Field Models?

Physical Reasons

- Link between the interplanetary medium and the ionosphere
- Guide and Energize charged particles
- Confines radiation belts and controls the auroral phenomena
- Store a huge amount of energy

Practical Reasons

- Satellite conjunctions (along magnetic field lines)
- Ground-based stations and spacecraft conjunctions
- Test particle simulations (Dr. Ganushkina)
- Mapping of ionospheric convection to the magnetosphere
- Subtract the background field from observation
Magnetic Field Models

- IGRF (International Geomagnetic Reference Field)
- Geopack (coordinate transformations, tilt ...)
- T89 (Tsyganenko, JGR, 1989)
  - Computationally fast
  - Extremely widely used
  - Parametrized by Kp
- T96
  - More realistic current systems
  - Larger set of input parameters
- All available at
Electric Field Models

  \[ \varphi = A r^\gamma \sin \phi, \text{ where } A \text{ is a constant, and } \gamma = 2, \text{ typically.} \]

Modelling Methods and Shortcomings

Magnetic Field

- Represent mathematically the field from each major current systems
- Represent the expected response of the field to physical factors that can be determined
- Calibrate the model against a database of average magnetic field observations, tagged by values of the solar wind parameters
- Force balance is not automatically built into the models
- Unreliable in highly time-variable situations such as substorms → event-oriented models (Deformation method below + Dr. Ganushkina)
Modelling Methods and Shortcomings (cont.)

3D Electric Field

- Equipotential mapping (static)
- \( \int_{\partial A} \mathbf{E} \cdot d\mathbf{l} = 0 \)
- Generalized mapping
- \( \int_{\partial A} \mathbf{E} \cdot d\mathbf{l} = - \int_A \partial_t \mathbf{B} \cdot d\mathbf{a} \)
- Induction included
- Computationally tedious

Deformation Method

Motivation

- Solving Faraday’s law is complicated ($\nabla \times E = -\partial_t B$)
- No explicit vector potential in magnetic field models
- Electrostatic component difficult to include
- $\partial_t A$ may have a component parallel to $B$
- Euler potentials → Physical electromagnetic fields

<table>
<thead>
<tr>
<th>Field</th>
<th>$(A, \phi)$</th>
<th>$(\alpha, \beta, \phi)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B$</td>
<td>$\nabla \times A$</td>
<td>$\nabla \alpha \times \nabla \beta$</td>
</tr>
<tr>
<td>$E_{vaccum}$</td>
<td>$-\partial_t A - \nabla \phi$</td>
<td>$-\partial_t (\alpha \nabla \beta) - \nabla \phi$</td>
</tr>
<tr>
<td>$E_{plasma}$</td>
<td>-</td>
<td>$-\partial_t \alpha \nabla \beta + \partial_t \beta \nabla \alpha - \nabla \phi$</td>
</tr>
</tbody>
</table>
Deformation Method (cont.)

- For a given $\alpha(x, y)$

- As to Homer, this can be done to a magnetic field:

  $B' = \nabla \hat{\alpha} \times \nabla \hat{\beta} \Rightarrow B' = T \hat{B}$

- Take a magnetic field given in some coordinates.

- Replace these coordinates by some transformed coordinates.

- Multiply the resulted magnetic field by the matrix $T$ (given on the next slide).

- The given field is then given in terms of the original coordinates

- References

Generalized Deformation Method

- Consider a transformation
  \( \hat{q}_i = f_i(q_1, q_2, q_3) \)
- Let \( \mathbf{A}' = \hat{\alpha} \nabla \hat{\beta} \)
  \( \Rightarrow \mathbf{A}' = \mathbf{M} \hat{\mathbf{A}}; \ M_{hl} = \frac{h_l}{h_h} \partial_h f_l \)
- Let \( [\nabla \phi]' = \nabla \hat{\phi} \)
  \( \Rightarrow [\nabla \phi]' = \mathbf{M}[\nabla \phi] \)
- Let \( \mathbf{B}' = \nabla \hat{\alpha} \times \nabla \hat{\beta} \)
  \( \Rightarrow \mathbf{B}' = \mathbf{T} \hat{\mathbf{B}}; \ T_{ij} = \frac{1}{2} \varepsilon_{ihk} M_{hl} M_{kq} \varepsilon_{lqj} \)
- Let \( \mathbf{E}' = -\partial_t \hat{\alpha} \nabla \hat{\beta} + \partial_t \hat{\beta} \nabla \hat{\alpha} - \nabla \hat{\phi} \)
  \( \Rightarrow \mathbf{E}' = \mathbf{M}(\hat{\mathbf{E}} + \partial_t \hat{\mathbf{q}} \times \hat{\mathbf{B}}) \)
- For deformed fields:
  - \( \mathbf{A}' \cdot \mathbf{B}' = 0 \)
  - \( \nabla \cdot \mathbf{B}' = 0 \)
  - \( \nabla \times \mathbf{B}' \neq 0, \text{ even if } \nabla \times \mathbf{B} = 0 \)
  - \( [\nabla \phi]' \cdot \mathbf{B}' = 0, \text{ if } \nabla \phi \cdot \mathbf{B} = 0 \)
  - \( \mathbf{E}' \cdot \mathbf{B}' = 0 \)
  - \( \nabla \times [\nabla \phi]' = 0 \)
  - \( \nabla \times \mathbf{E}' = -\partial_t \mathbf{B}' \)

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Initial Dipole Fields

□ In spherical coordinates \((r, \theta, \varphi)\):

□ Euler potentials: \(\alpha \propto r^{-1} \sin^2 \theta\) and \(\beta \propto \varphi\)

\[ B = \frac{\mu_0 m}{2\pi r^3} \cos \theta e_r + \frac{\mu_0 m}{4\pi r^3} \sin \theta e_\theta \]

\[ A = \frac{\mu_0 m}{4\pi r^2} \sin \theta e_\varphi \]

□ Assume equipotential dipole field lines: \(-\nabla \phi \cdot B = 0\)

\[ \phi(r, \theta, \varphi) = \phi_{is}(r_{is}, \theta_{is}(r, \theta), \varphi_{is}(\varphi)) \]

□ Dipole magnetic field lines

\[ (r_{is}, \sin \theta_{is}, \varphi_{is}) = (\text{constant}, \left[\frac{r_{is}}{r}\right]^{\frac{1}{2}} \sin \theta, \varphi) \]

□ Assume an ionospheric electric field \(E^{is} = (E^i_{\theta}, E^i_{\varphi})\)

\[ E = -\nabla \phi = -\nabla \phi_{is} = \left(\frac{r_{is}}{r}\right)^{\frac{3}{2}} \left(-\frac{1}{2} \frac{\sin \theta}{\cos \theta_{is}} E^i_{\theta} e_r + \frac{\cos \theta}{\cos \theta_{is}} E^i_{\theta} e_\theta + E^i_{\varphi} e_\varphi \right) \]
An Example: Dipole Expansion

- Dipole field (dash-dotted)
- \( \alpha = \frac{\mu_0 m_0}{4\pi r} \sin^2 \theta \)
- \( m \rightarrow m(r) \)
- \( \hat{r} = r H(r)^{-1} \)
- \( H(r) = a \left( 1 - \tanh \left( \frac{r - r_o}{\Delta r} \right) \right) + c \)
- \( a = (1 - c) \left( 1 - \tanh \left( \frac{r_{is} - r_o}{\Delta r} \right) \right)^{-1} \)
- \( m = m_o, \quad \text{if} \quad r = r_{is} \)
- \( m = cm_o, \quad \text{if} \quad r \rightarrow \infty \)
- Expanded dipole (solid)
- \( \Rightarrow \) Ring current
Model Construction

Noon-midnight meridian

- Consecutive deformations:
- Dipole expansion (a)
- Field topology (b)
- Near-Earth plasma sheet (c)
- Dayside compression (d)
- Magnetopause and lobes (e)
- Tilt effect (f)
Model Construction (cont.)

Equatorial plane

- Last closed field lines:
- Dipole expansion (a)
- Field topology (b)
- Near-Earth plasma sheet (c)
- Dayside compression (d)
- Magnetopause and lobes (e)
SSC, April 06, 2000

- Solar wind pressure pulse
- Negative excursion of IMF $B_Z$
- Dayside compression at 1640 UT

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Magnetic Field

- GOES 8 located at noon close to the magnetic equator (see next slide)
- Measured field at GOES 8 (black)
- Modelled field (red)
- Magnetic field compression ($B_Z^{GSM}$)

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Magnetic Field (cont.)

- Polar located at noon close to the northern cusp (see next slide)
- Measured field at Polar (black)
- MHD model from GU-MICS (blue)
- Magnetic field compression ($B_X^{\text{GSM}}$)
- Large deflection of $B_Y^{\text{GSM}}$
Magnetic Field Configurations

1630 UT Polar(△), GOES8 (◇)

1650 UT
Electric Field Configurations

- Static electric field at 1630 UT
- Corresponds to the mapped ionospheric convection
- Determined by deformation to the initial dipole potential field
- $\triangle = \text{Polar}$, $\diamond = \text{GOES 8}$
Electric Field Configurations (cont.)

- Static electric field at 1650 UT
- $\triangle = \text{Polar, } \diamond = \text{GOES 8}$
- Field close to the Earth is due corotation (plasmashpere)
Electric Field Configurations (cont.)

- Induced electric field at 1640 UT
- Corresponds to the field lines motion
- Due to time-evolution of $B_Y^{\text{GSM}}$
Electric Field at Polar

Convection electric field, Polar (black), modelled (red), GUMICS (blue)

$E_{X}^{GSE}$

$E_{Z}^{GSE}$
Electric Field at Polar (cont.)

Induced electric field, Polar (black), modelled (red), GUMICS (blue)
Electric Field at Polar (cont.)

Total electric field, Polar (black), modelled (red), GUMICS (blue)