Electromagnetic Fields Inside the Magnetoshpere

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Outline

- Introduction to large-scale electromagnetic fields
  - Magnetic field geometry
  - Magnetospheric convection
  - Magnetospheric current systems
  - Magnetospheric dynamics
- Motivation
- Magnetic field models
  - Historical models
  - Present-day models
- Electric field models
  - Equatorial models
  - Ionospheric models
- Modelling methods
  - $\nabla \times \mathbf{B} = \mu_o \mathbf{J}$
  - $\mathbf{E} \cdot \mathbf{B} = 0$
- Deformation method
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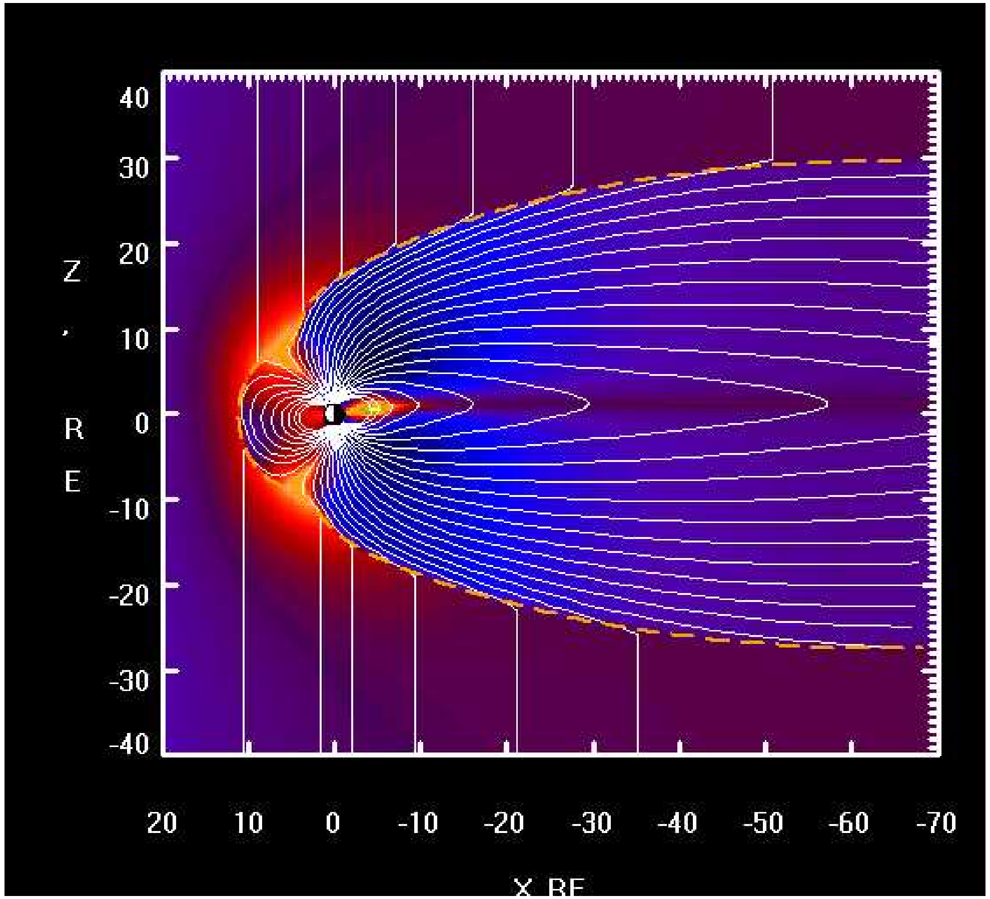
  - Euler potentials
  - Theory
  - Example & Model construction
  - 3D electric fields
- Modelling example
  - SSC, April 06, 2000
  - Dayside compression

Intro to Large-Scale Electromagnetic Fields

Magnetic Field Geometry

- Internal magnetic field
- Dipole tilt
- Solar wind pressure
- Open field lines
- Closed field lines
- Cusp

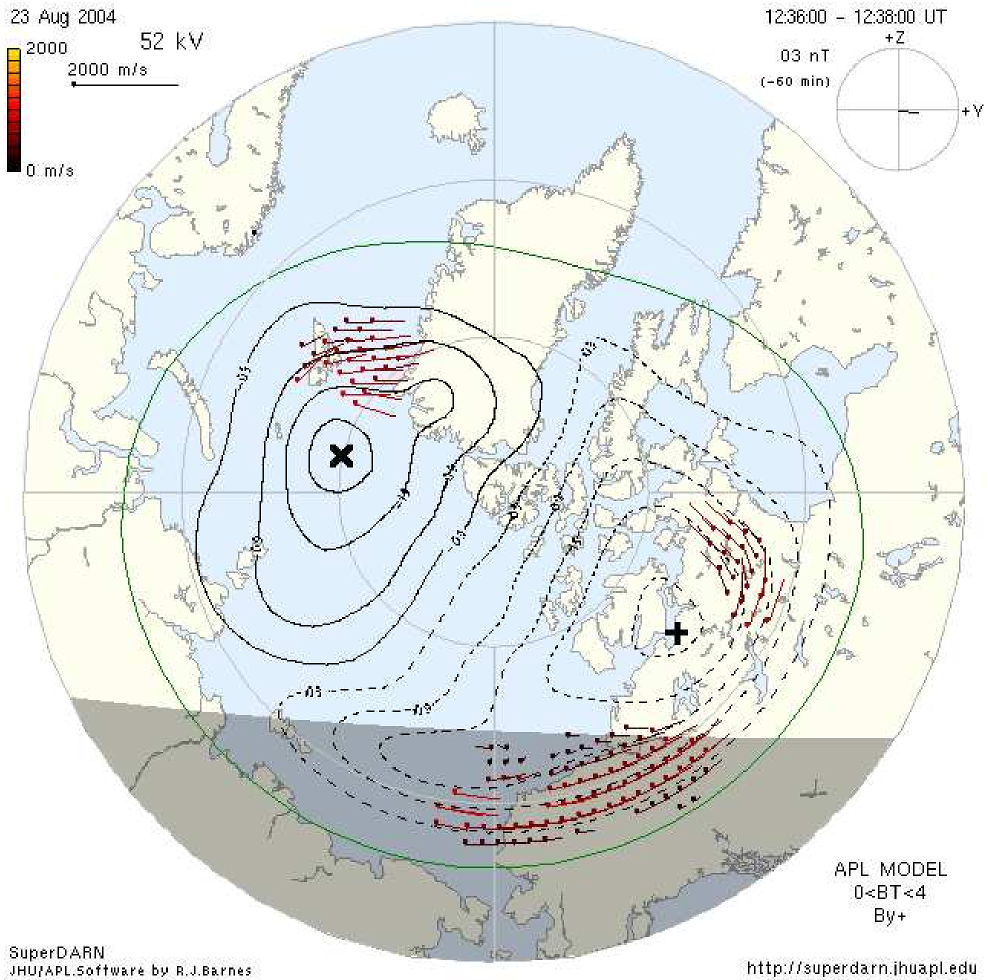


http://modelweb.gsfc.nasa.gov/magnetos/data-based/modeling.html

Intro to Large-Scale Electromagnetic Fields (cont.)

Convection

- Ionosphere:
- Global convection maps to the ionospheric convection
- Typically two-shell pattern (IMF $B_Z < 0$)
- High-latitude convection from noon to midnight
- Return convection at lower latitudes via dawn and dusk
- Can be measured by radars

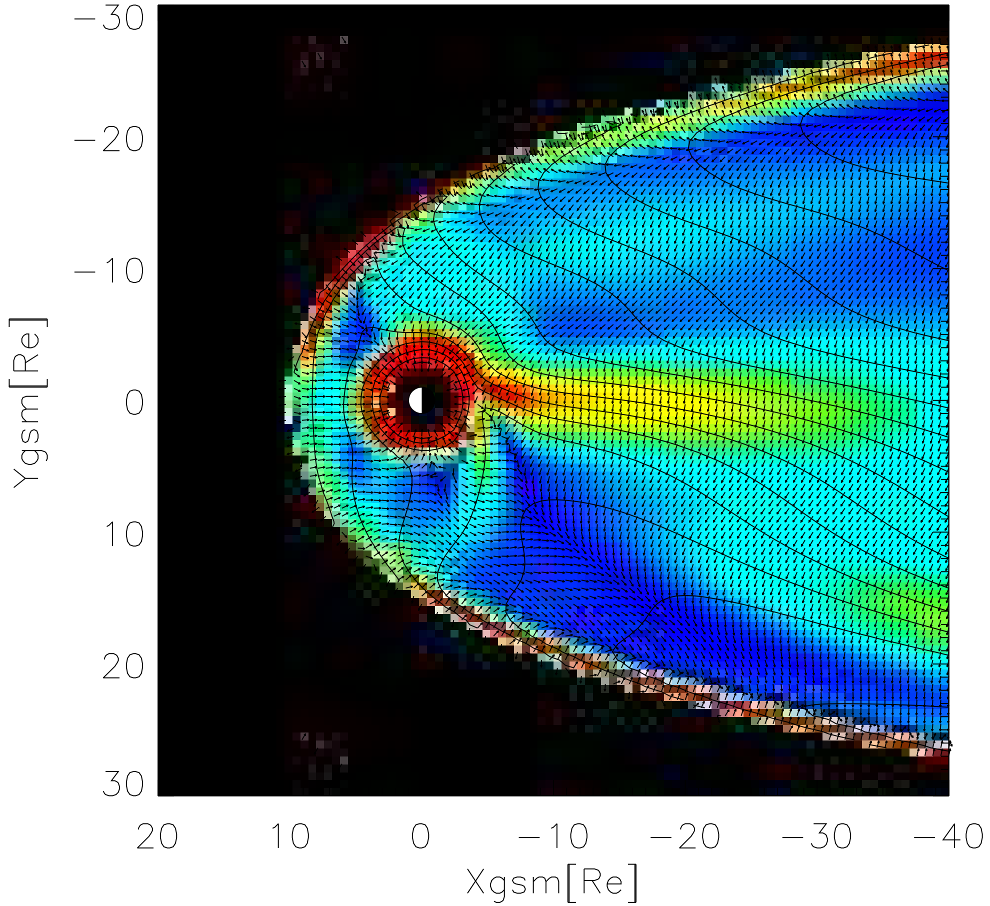


http://superdarn.jhuapl.edu/

Intro to Large-Scale Electromagnetic Fields (cont.)

Convection (cont.)

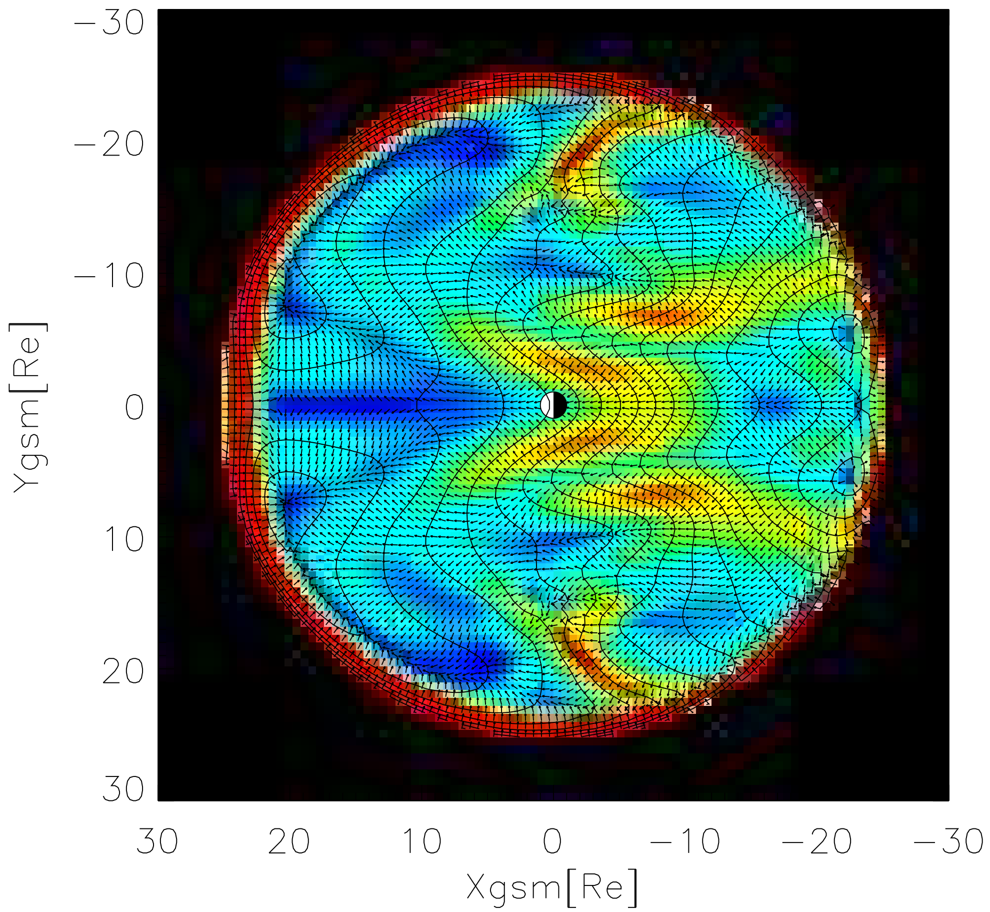
- Equatorial plane:
- Earthward convection in the mid-tail
- Plasma flow around the co-rotating plasmasphere to the dayside
- Return convection along the flanks
- Electric field associated with the convection ~ 0.5mV/m
- Cannot be globally measured



Intro to Large-Scale Electromagnetic Fields (cont.)

Convection (cont.)

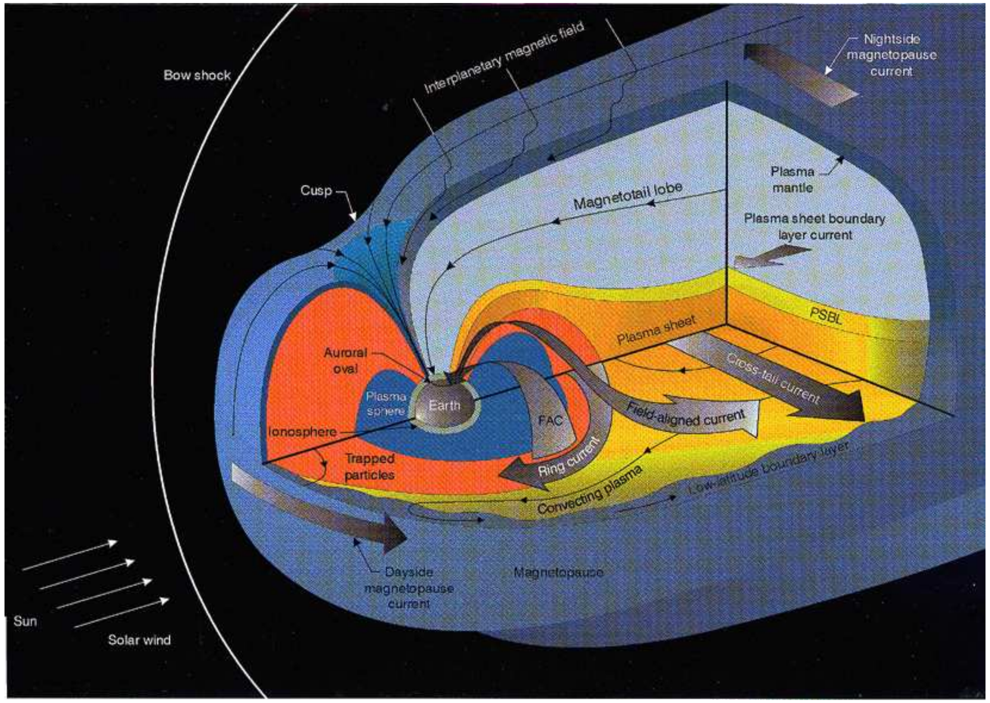
- Tail cross-section:
- Convection across the tail lobes towards the plasma sheet
- Turns to earthward convection at the plasma sheet



Intro to Large-Scale Electromagnetic Fields (cont.)

Current Systems

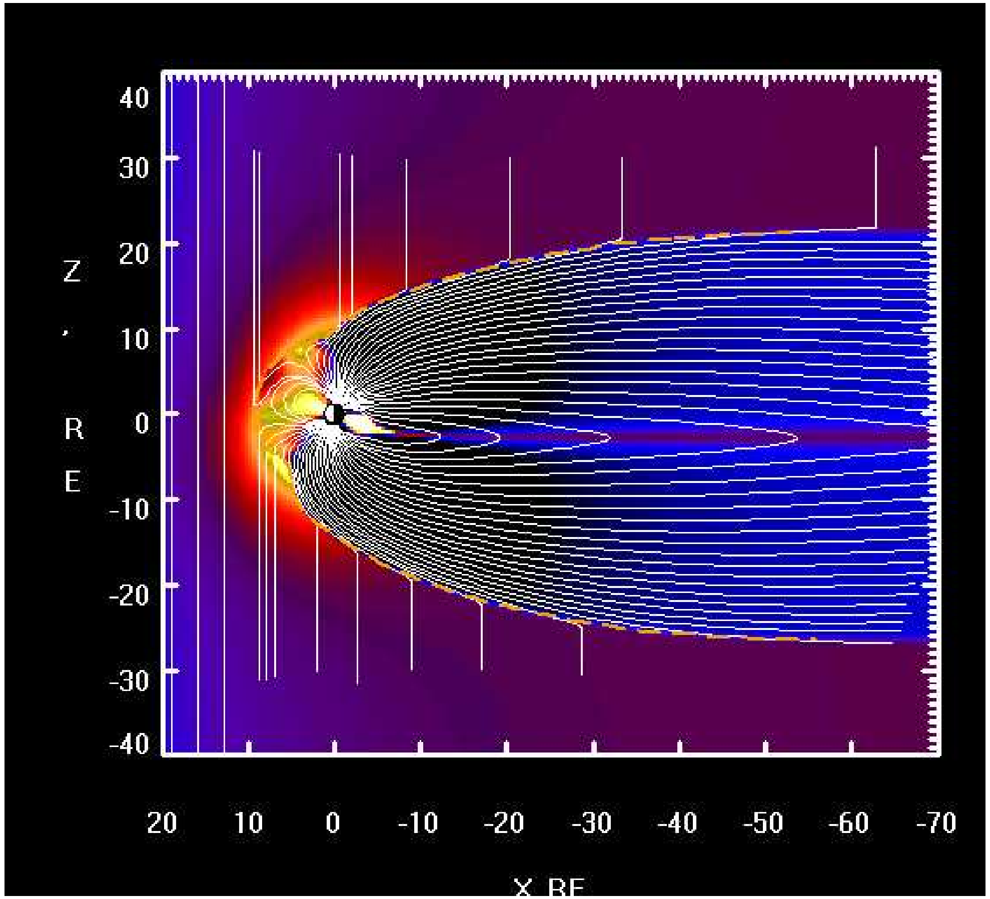
- Cross-tail current
- Ring current
- Magnetopause current
- Field-aligned currents



Intro to Large-Scale Electromagnetic Fields (cont.)

Magnetospheric Dynamics

- Magnetic storms
- Magnetopause compression
- Ring current enhancement
- Substorms
- Tail stretching
- Dipolarization



http://modelweb.gsfc.nasa.gov/magnetos/data-based/modeling.html

Why Analytical Field Models?

Physical Reasons

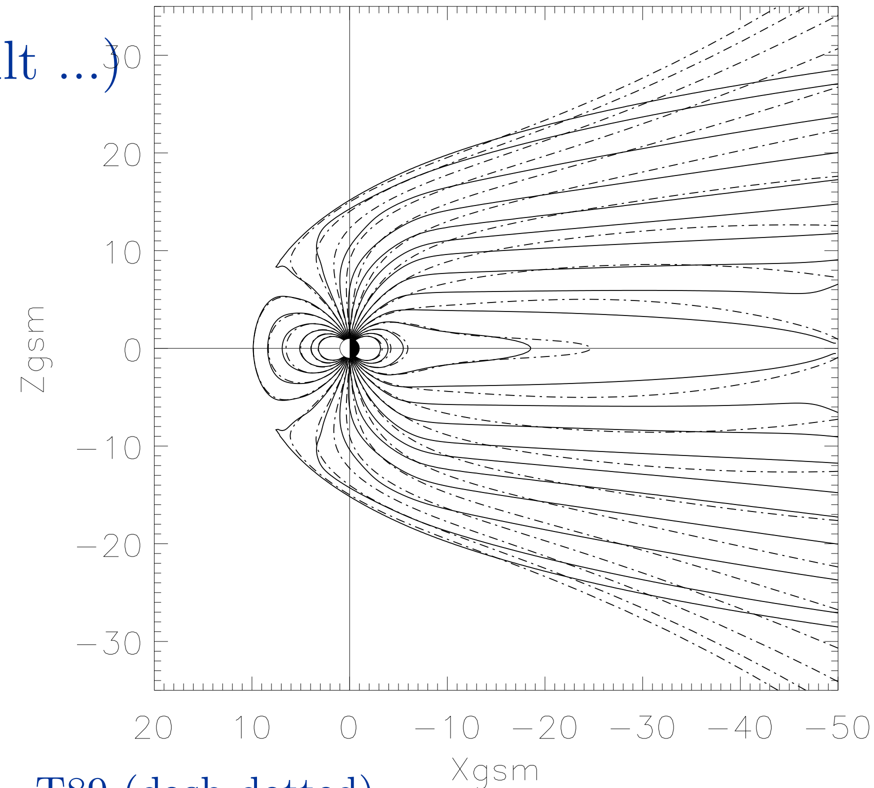
- Link between the interplanetary medium and the ionosphere
- Guide and Energize charged particles
- Confines radiation belts and controls the auroral phenomena
- Store a huge amount of energy

Practical Reasons

- Satellite conjunctions (along magnetic field lines)
- Ground-based stations and spacecraft conjunctions
- Test particle simulations (Dr. Ganushkina)
- Mapping of ionospheric convection to the magnetosphere
- Subtract the backround field from observation

Magnetic Field Models

- For historical models: See Stern, JGR, 1994.
- IGRF (International Geomagnetic Reference Field)
- Geopack (coordinate transformations, tilt ...)
- T89 (Tsyganenko, JGR, 1989)
  - ▷ Computationally fast
  - ▷ Extremely widely used
  - ▷ Parametrized by Kp
- T96
  - ▷ More realistic current systems
  - ▷ Larger set of input parameters
- All available at http://modelweb.gsfc.nasa.gov/magnetos/data-based/modeling.html

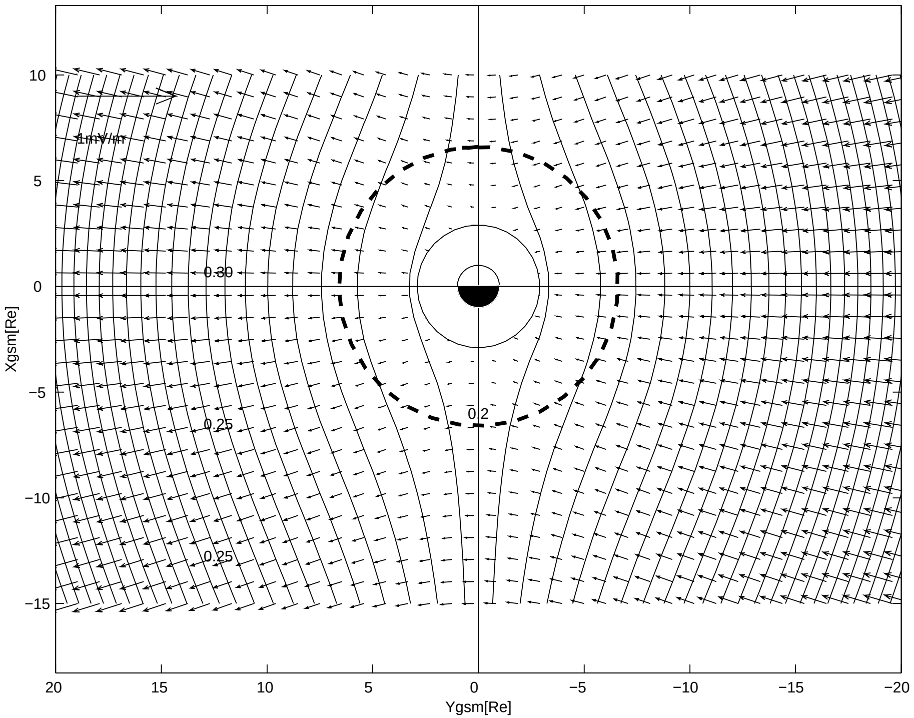


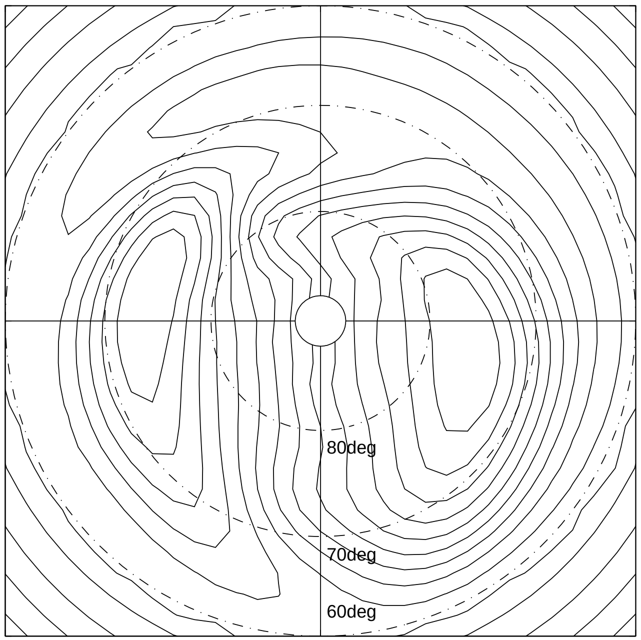
T89 (dash-dotted)

Toivanen (solid)

Electric Field Models

- Equatorial model: Volland, JGR, 1973.
  - ▷ $\varphi = Ar^{\gamma} \sin \phi$, where A is a constant, and $\gamma = 2$, typically.
- Ionospheric model: Heppner and Maynard, JGR, 1987.





Modelling Methods and Shortcomings

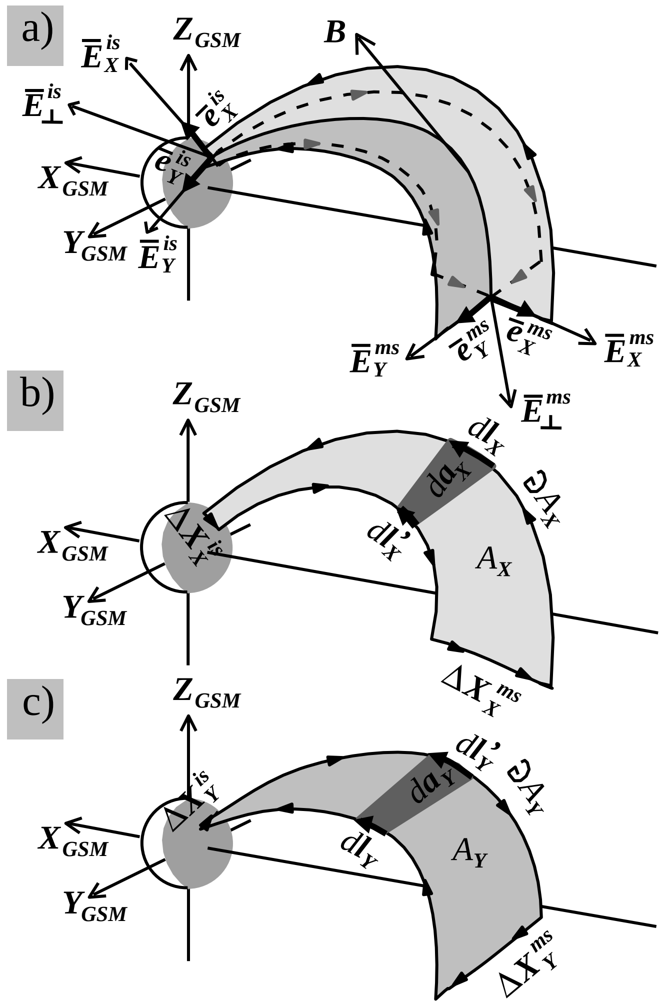
Magnetic Field

- Represent mathematically the field from each major current systems
- Represent the expected response of the field to physical factors thatcan be determined
- Calibrate the model against a database of average magnetic field observations, tagged by values of the solar wind parameters
- Force balance is not automatically built into the models
- Unreliable in highly time-variable situations such as substorms $\rightarrow$ event-oriented models (Deformation method below + Dr. Ganushkina)

Modelling Methods and Shortcomings (cont.)

3D Electric Field

- Equipotential mapping (static)
- $\int_{\partial A} \mathbf{E} \cdot \mathbf{dl} = 0$
- Generalized mapping
- $\int_{\partial A} \mathbf{E} \cdot \mathbf{dl} = -\int_A \partial_t \mathbf{B} \cdot \mathbf{da}$
- Induction included
- Computationally tedious
- Toivanen, P. K., et al., JGR, 1998.



Deformation Method

Motivation

- Solving Faraday's law is complicated ($\nabla \times \mathbf{E} = -\partial_t \mathbf{B}$)
- No explicit vector potential in magnetic field models
- Electrostatic component difficult to include
- $\partial_t \mathbf{A}$ may have a component parallel to $\mathbf{B}$
- Euler potentials $\rightarrow$ Physical electromagnetic fields

Field	$(\mathbf{A}, \phi)$	(α, β, ϕ)
$\mathbf{B}$	$\nabla \times \mathbf{A}$	$\nabla\alpha \times \nabla\beta$
$\mathbf{E}_{vacuum}$	$-\partial_t \mathbf{A} - \nabla\phi$	$-\partial_t(\alpha\nabla\beta) - \nabla\phi$
$\mathbf{E}_{plasma}$	-	$-\partial_t\alpha\nabla\beta + \partial_t\beta\nabla\alpha - \nabla\phi$

Deformation Method (cont.)

□ For a given $\alpha(x, y)$

□ $\widehat{\alpha}(x, y) = \alpha(\widehat{x}, \widehat{y})$

- As to Homer, this can be done to a magnetic field:
- $\mathbf{B}' = \nabla\widehat{\alpha} \times \nabla\widehat{\beta} \;\Rightarrow \mathbf{B}' = \mathbf{T}\widehat{\mathbf{B}}$
- Take a magnetic field given in some coordinates.
- Replace these coordinates by some transformed coordinates.
- Multiply the resulted magnetic field by the matrix $\mathbf{T}$ (given on the next slide).
- The given field is then given in terms of the original coordinates
- References
  - ▷ Stern, D. P., JGR, 1987.
  - ▷ Tsyganenko, N. A., JGR, 1998.

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Generalized Deformation Method

- Consider a transformation

$\widehat{q}_i = f_i(q_1, q_2, q_3)$

- Let $\mathbf{A}' = \widehat{\alpha}\nabla\widehat{\beta}$

$\Rightarrow \mathbf{A}' = \mathbf{M}\widehat{\mathbf{A}};\ M_{hl} = \frac{\widehat{h}_l}{h_h}\partial_h f_l$

- Let $[\nabla\phi]' = \nabla\widehat{\phi}$

$\Rightarrow [\nabla\phi]' = \mathbf{M}[\widehat{\nabla\phi}]$

- Let $\mathbf{B}' = \nabla\widehat{\alpha} \times \nabla\widehat{\beta}$

$\Rightarrow \mathbf{B}' = \mathbf{T}\widehat{\mathbf{B}};\ T_{ij} = \frac{1}{2}\varepsilon_{ihk}M_{hl}M_{kq}\varepsilon_{lqj}$

- Let $\mathbf{E}' = -\partial_t\widehat{\alpha}\nabla\widehat{\beta} + \partial_t\widehat{\beta}\nabla\widehat{\alpha} - \nabla\widehat{\phi}$

$\Rightarrow \mathbf{E}' = \mathbf{M}(\widehat{\mathbf{E}} + \partial_t\widehat{\mathbf{q}} \times \widehat{\mathbf{B}})$

- For deformed fields:
- $\mathbf{A}' \cdot \mathbf{B}' = 0$
- $\nabla \cdot \mathbf{B}' = 0$
- $\nabla \times \mathbf{B}' \neq 0$, even if $\nabla \times \mathbf{B} = 0$
- $[\nabla\phi]' \cdot \mathbf{B}' = 0$, if $\nabla\phi \cdot \mathbf{B} = 0$
- $\mathbf{E}' \cdot \mathbf{B}' = 0$
- $\nabla \times [\nabla\phi]' = 0$
- $\nabla \times \mathbf{E}' = -\partial_t\mathbf{B}'$

Initial Dipole Fields

□ In spherical coordinates (r, θ, φ):

□ Euler potentials: $\alpha \propto r^{-1} \sin^2 \theta$ and $\beta \propto \varphi$

$\Rightarrow \mathbf{B} = \frac{\mu_o m}{2\pi r^3} \cos\theta \mathbf{e}_r + \frac{\mu_o m}{4\pi r^3} \sin\theta \mathbf{e}_\theta$

$\Rightarrow \mathbf{A} = \frac{\mu_o m}{4\pi r^2} \sin\theta \mathbf{e}_\varphi$

□ Assume equipotential dipole field lines: $-\nabla\phi \cdot \mathbf{B} = 0$

$\Rightarrow \phi(r, \theta, \varphi) = \phi_{is}(r_{is}, \theta_{is}(r, \theta), \varphi_{is}(\varphi))$

□ Dipole magnetic field lines

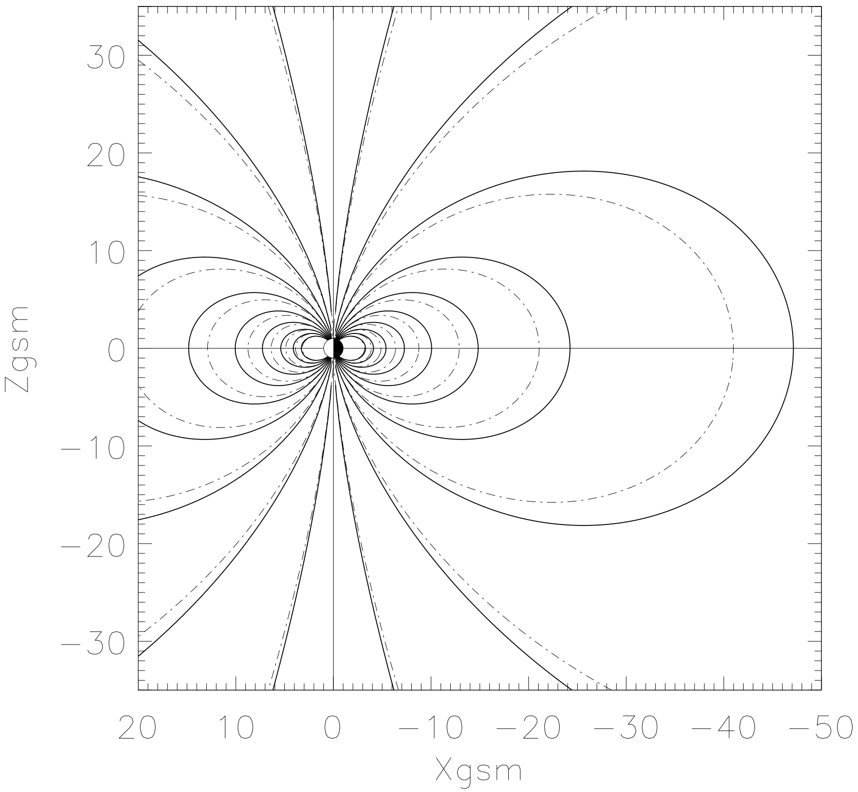
$\Rightarrow (r_{is}, \sin\theta_{is}, \varphi_{is}) = (\text{constant}, \left[\frac{r_{is}}{r}\right]^{\frac{1}{2}} \sin\theta, \varphi)$

□ Assume an ionospheric electric field $\mathbf{E}^{is} = (E_\theta^{is}, E_\varphi^{is})$

$\Rightarrow \mathbf{E} = -\nabla\phi = -\nabla\phi_{is} = \left(\frac{r_{is}}{r}\right)^{\frac{3}{2}} (-\frac{1}{2}\frac{\sin\theta}{\cos\theta_{is}} E_\theta^{is} \mathbf{e}_r + \frac{\cos\theta}{\cos\theta_{is}} E_\theta^{is} \mathbf{e}_\theta + E_\varphi^{is} \mathbf{e}_\varphi)$

An Example: Dipole Expansion

- Dipole field (dash-dotted)
- $\alpha = \frac{\mu_o m_o}{4\pi r} \sin^2\theta$
- $m \rightarrow m(r)$
- $\hat{r} = rH(r)^{-1}$
- $H(r) = a\left(1 - \tanh\left(\frac{r-r_o}{\Delta r}\right)\right) + c$
- $a = (1-c)\left(1 - \tanh\left(\frac{r_{\rm is}-r_o}{\Delta r}\right)\right)^{-1}$
- $m = m_o$, if $r = r_{\rm is}$
- $m = cm_o$, if $r \rightarrow \infty$
- Expanded dipole (solid)
- $\Rightarrow$ Ring current



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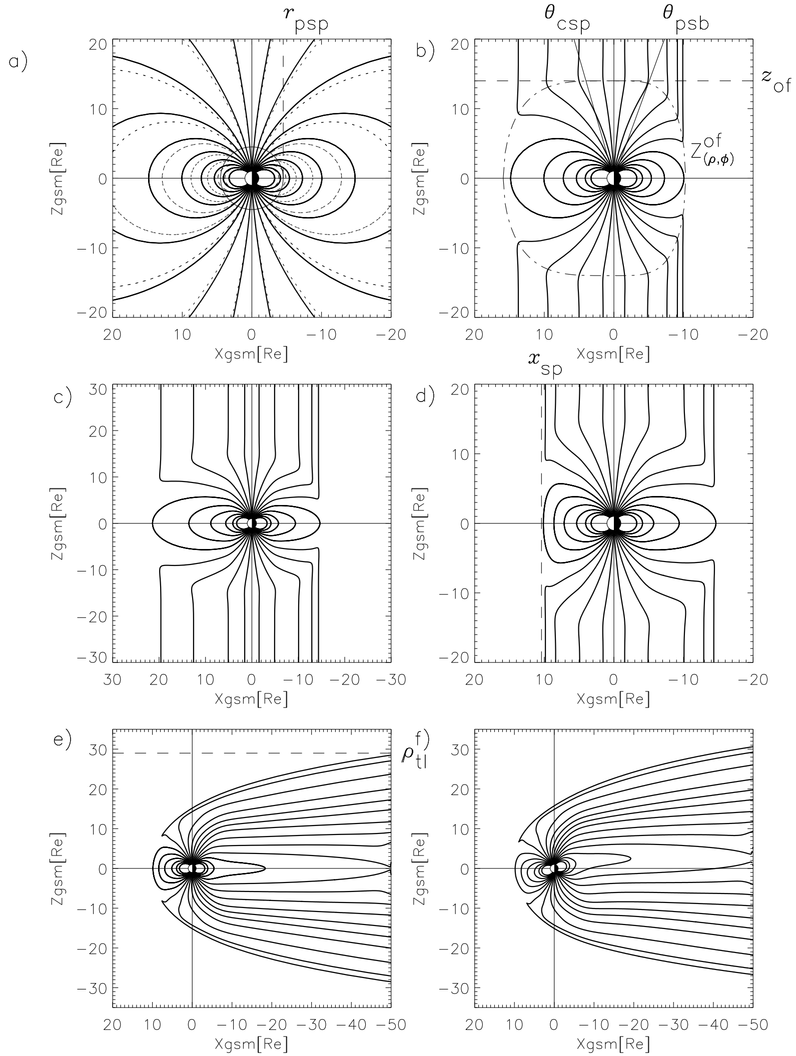
Model Construction

Noon-midnight meridian

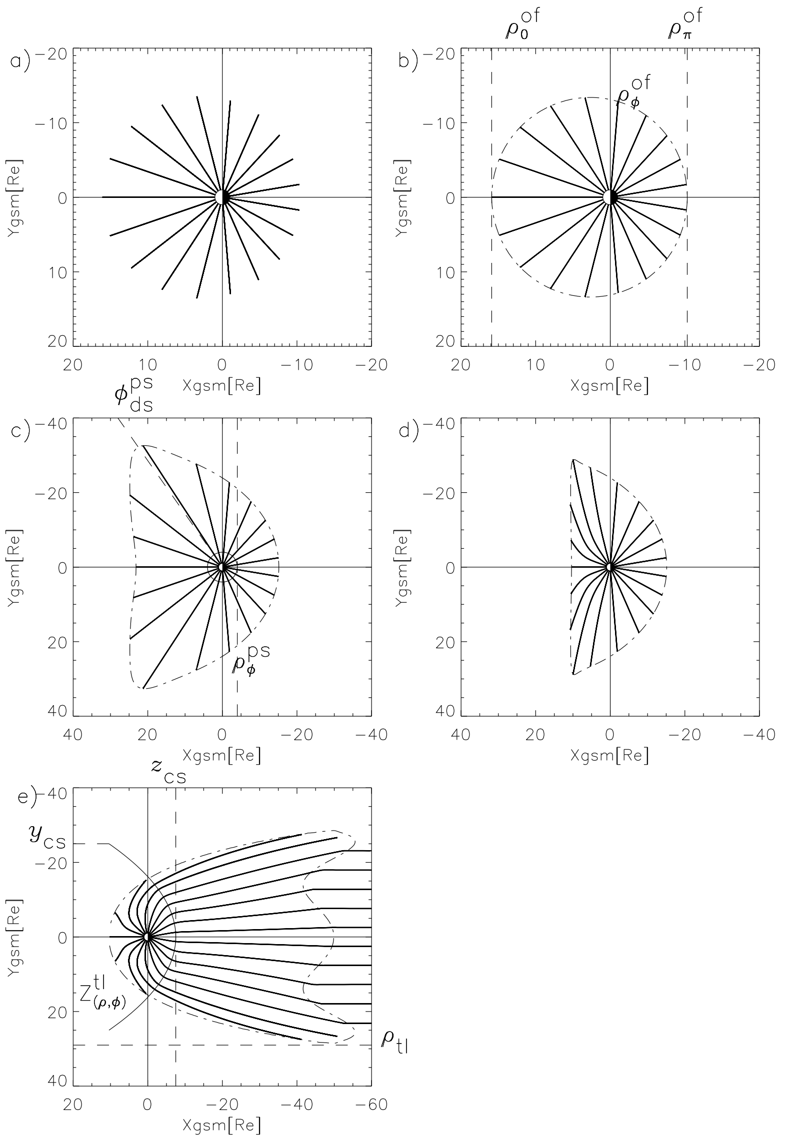
- Consecutive deformations:
- Dipole expansion (a)
- Field topology (b)
- Near-Earth plasma sheet (c)
- Dayside compression (d)
- Magnetopause and lobes (e)
- Tilt effect (f)



Model Construction (cont.)

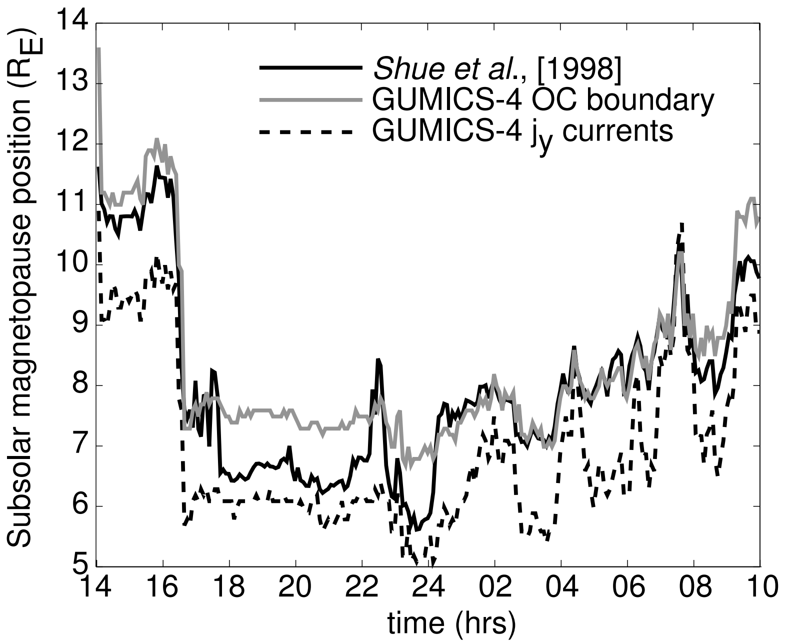
Equatorial plane

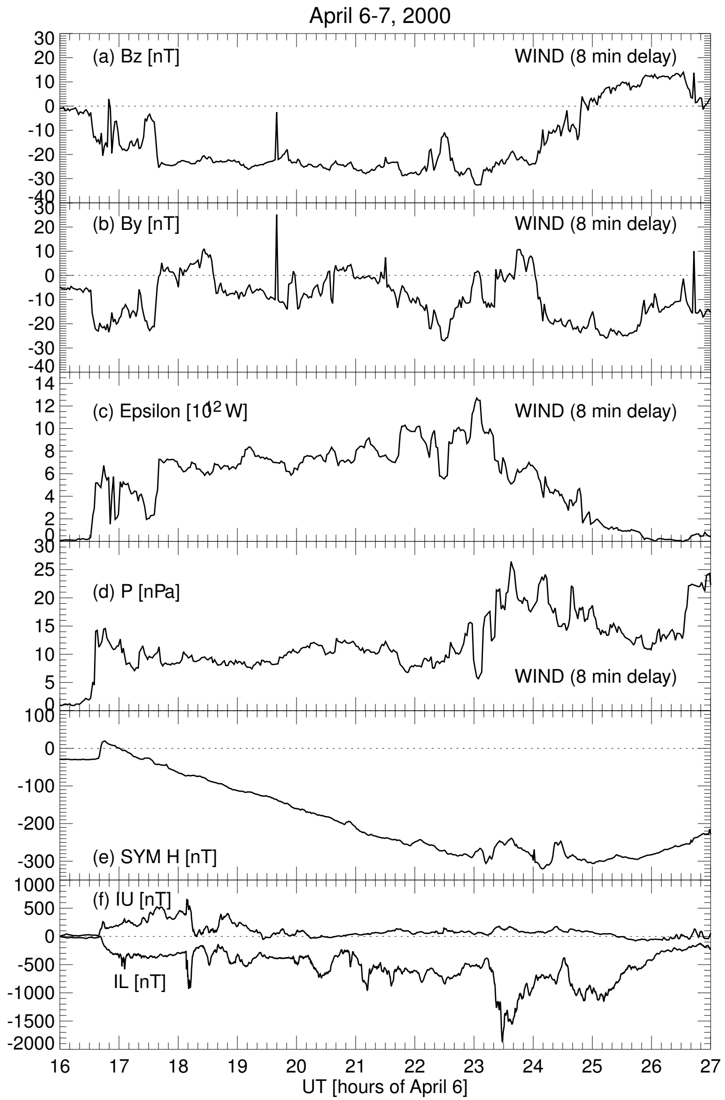
- Last closed field lines:
- Dipole expansion (a)
- Field topology (b)
- Near-Earth plasma sheet (c)
- Dayside compression (d)
- Magnetopause and lobes (e)



SSC, April 06, 2000

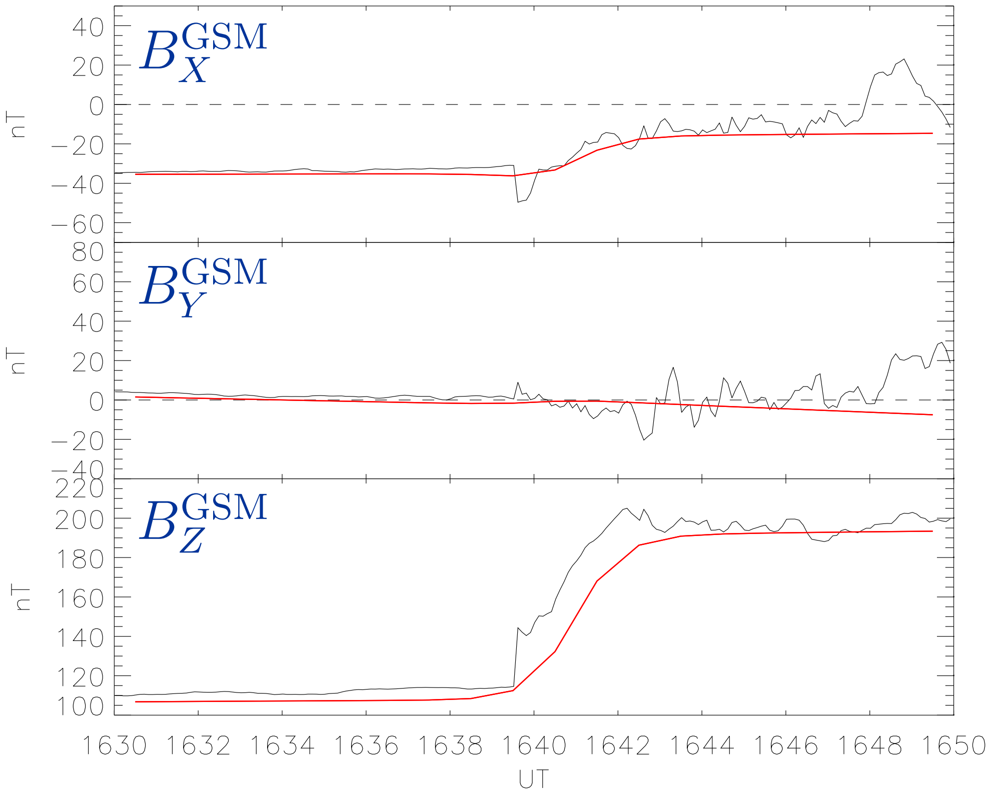
- Solar wind pressure pulse
- Negative excursion of IMF B_Z
- Dayside compression at 1640 UT





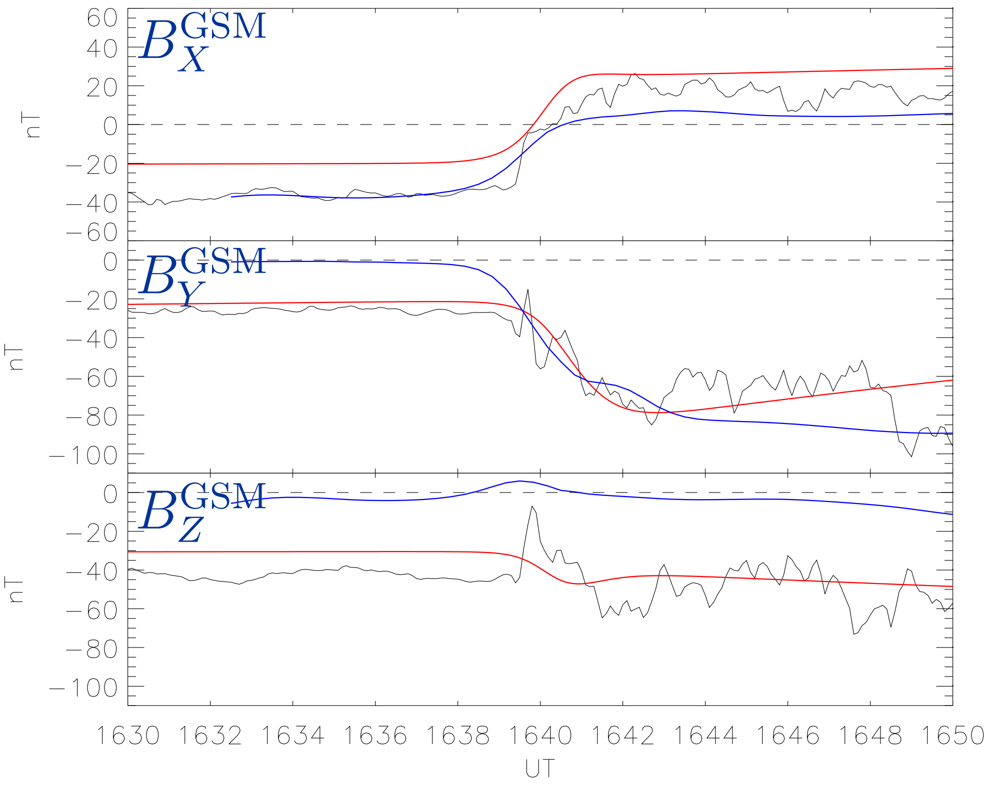
Magnetic Field

- GOES 8 located at noon close to the magnetic equator (see next slide)
- Measured field at GOES 8 (black)
- Modelled field (red)
- Magnetic field compression (B_Z^{GSM})



Magnetic Field (cont.)

- Polar located at noon close to the northern cusp (see next slide)
- Measured field at Polar (black)
- MHD model from GUMICS (blue)
- Magnetic field compression (B_X^{GSM})
- Large deflection of B_Y^{GSM}



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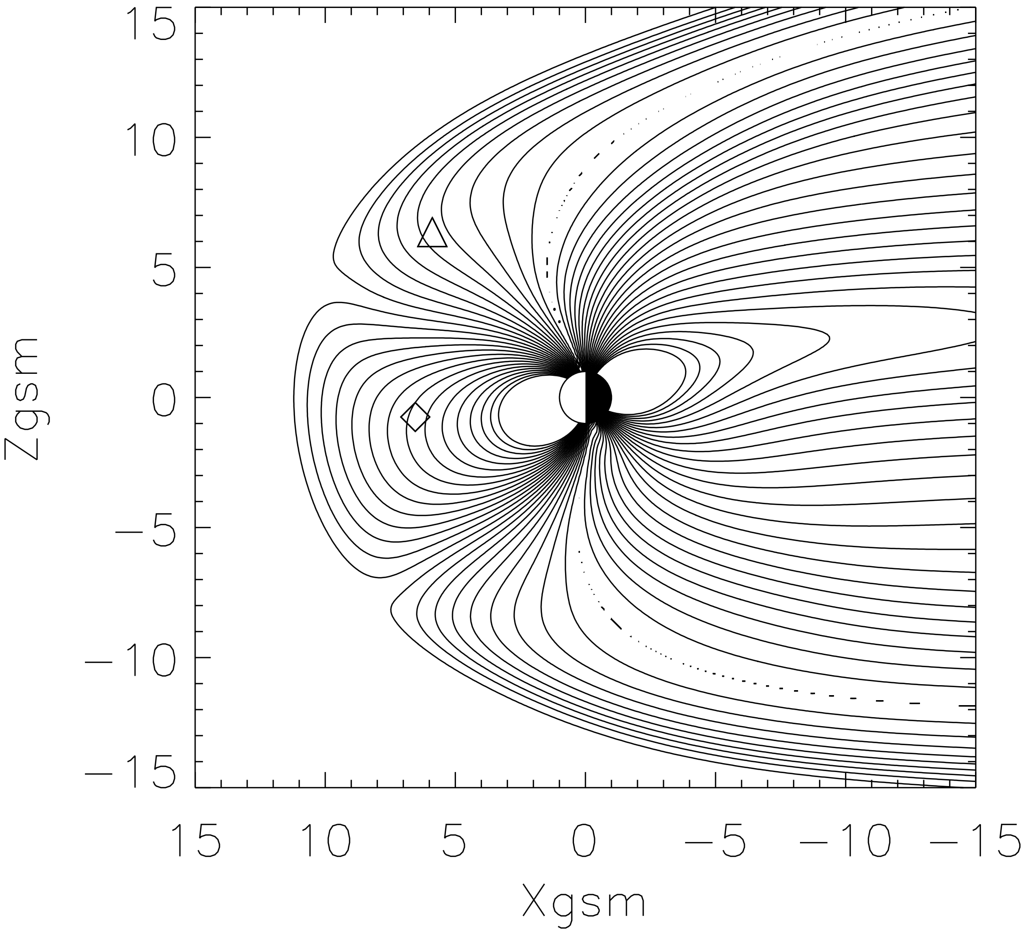
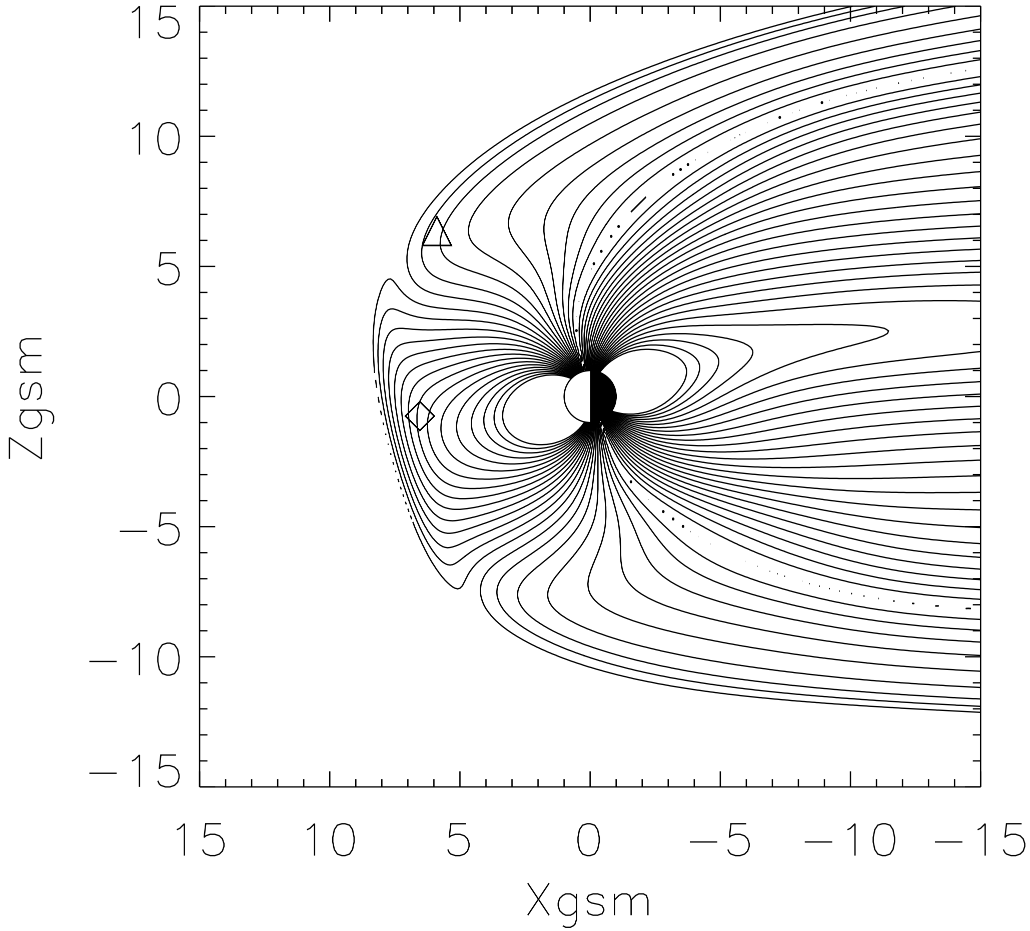
Magnetic Field Configurations

1630 UT Polar(△), GOES8 (◇)

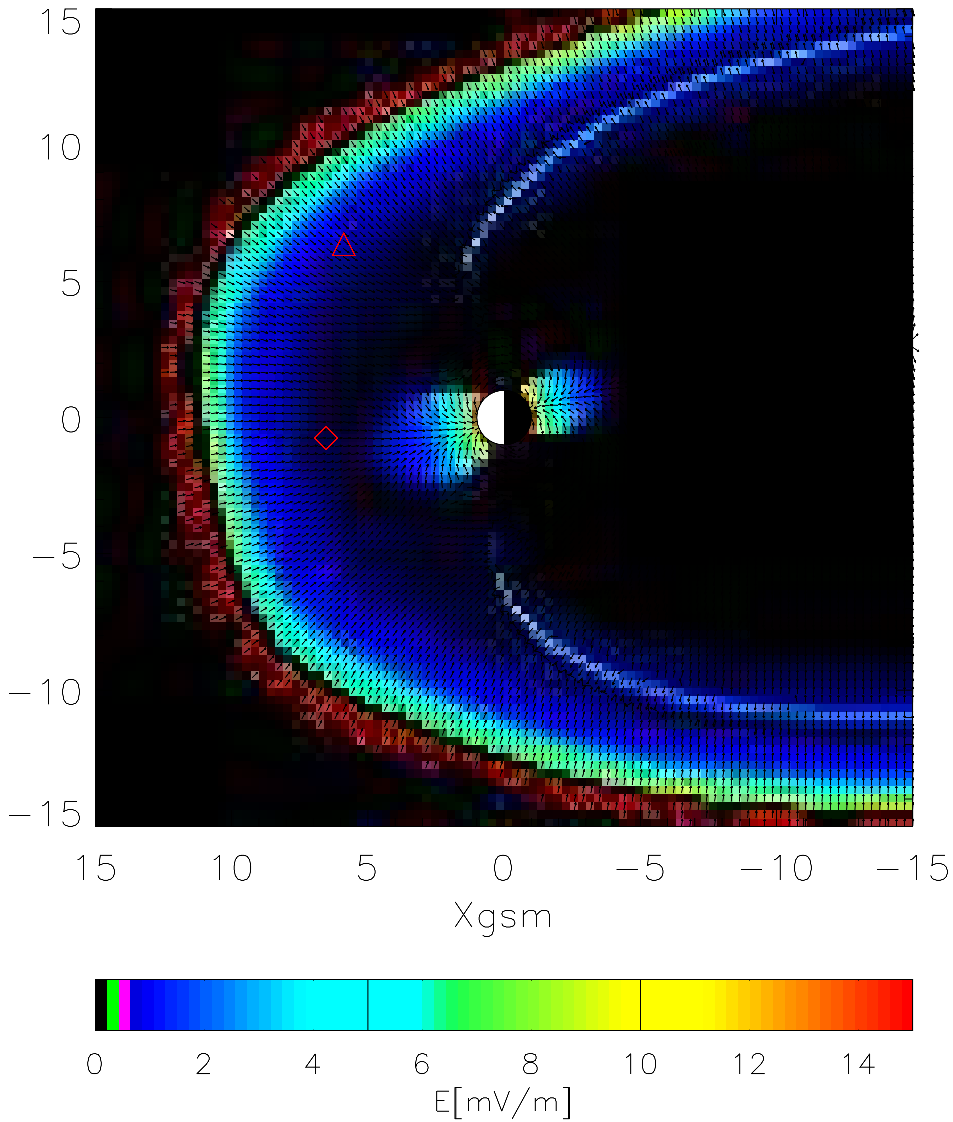


1650 UT



Electric Field Configurations

- Static electric field at 1630 UT
- Corresponds to the mapped ionospheric convection
- Determined by deformation to the initial dipole potential field
- $\triangle$ = Polar, $\diamond$ = GOES 8



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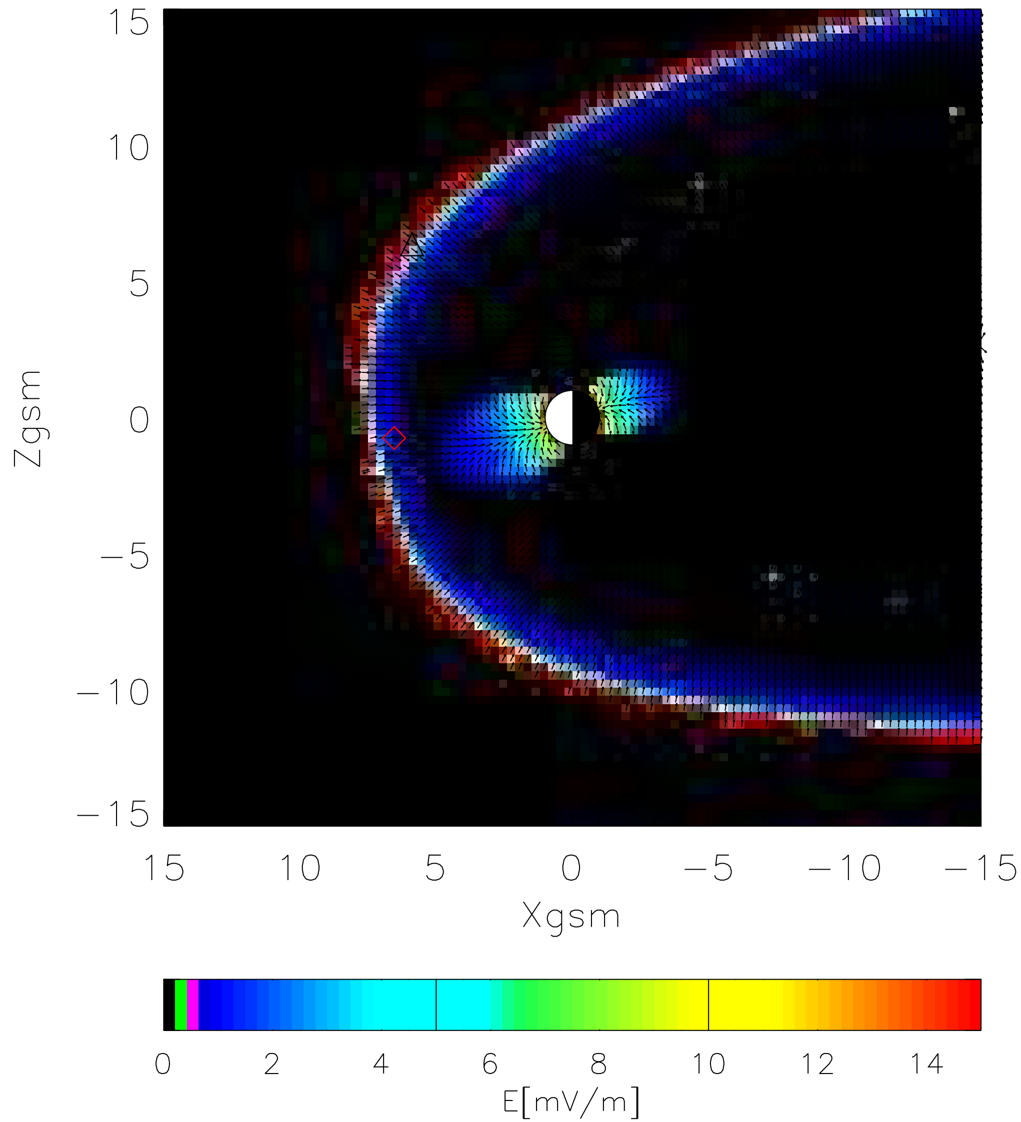
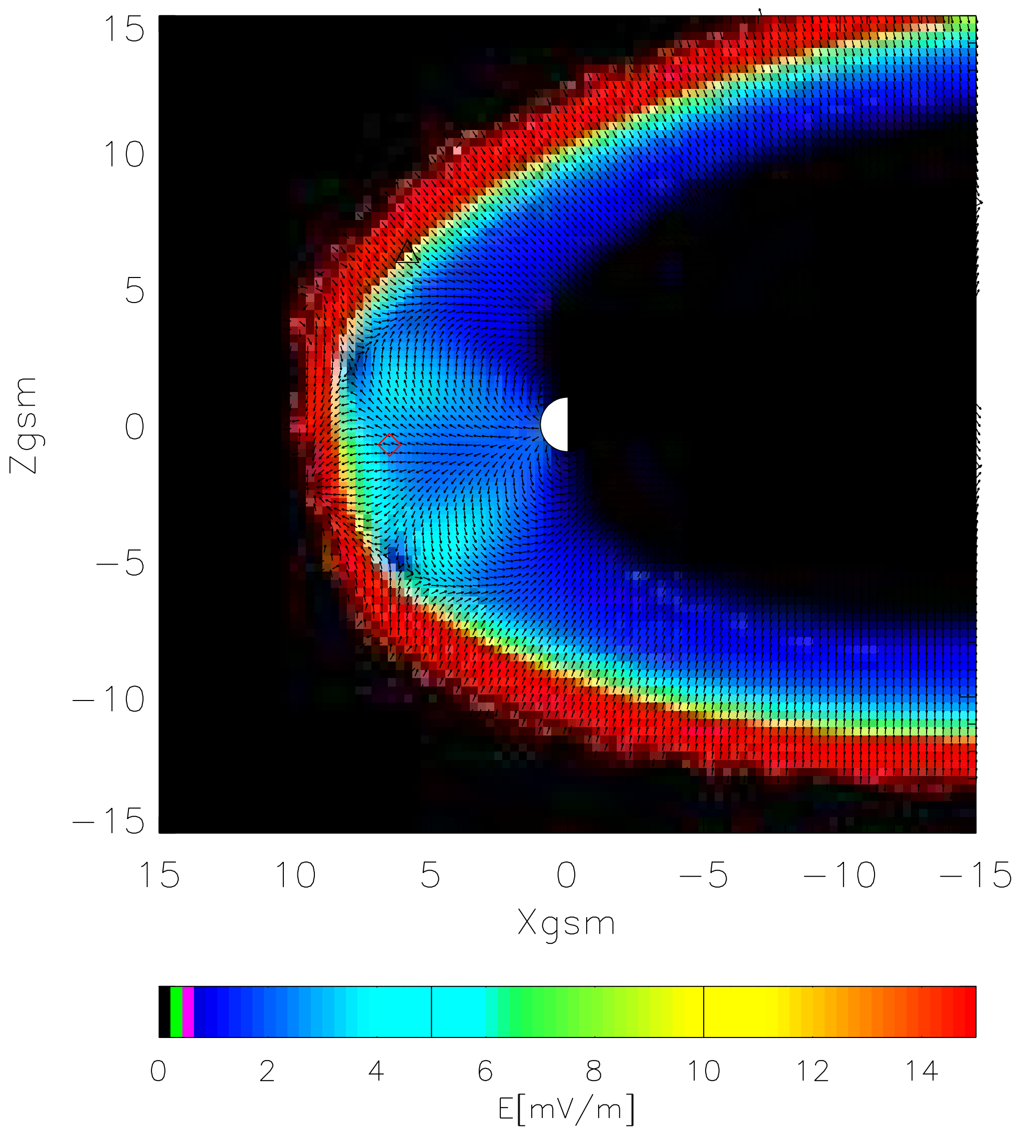
Electric Field Configurations (cont.)

- Static electric field at 1650 UT
- $\triangle$ = Polar, $\diamond$ = GOES 8
- Field close to the Earth is due corotation (plasmashpere)



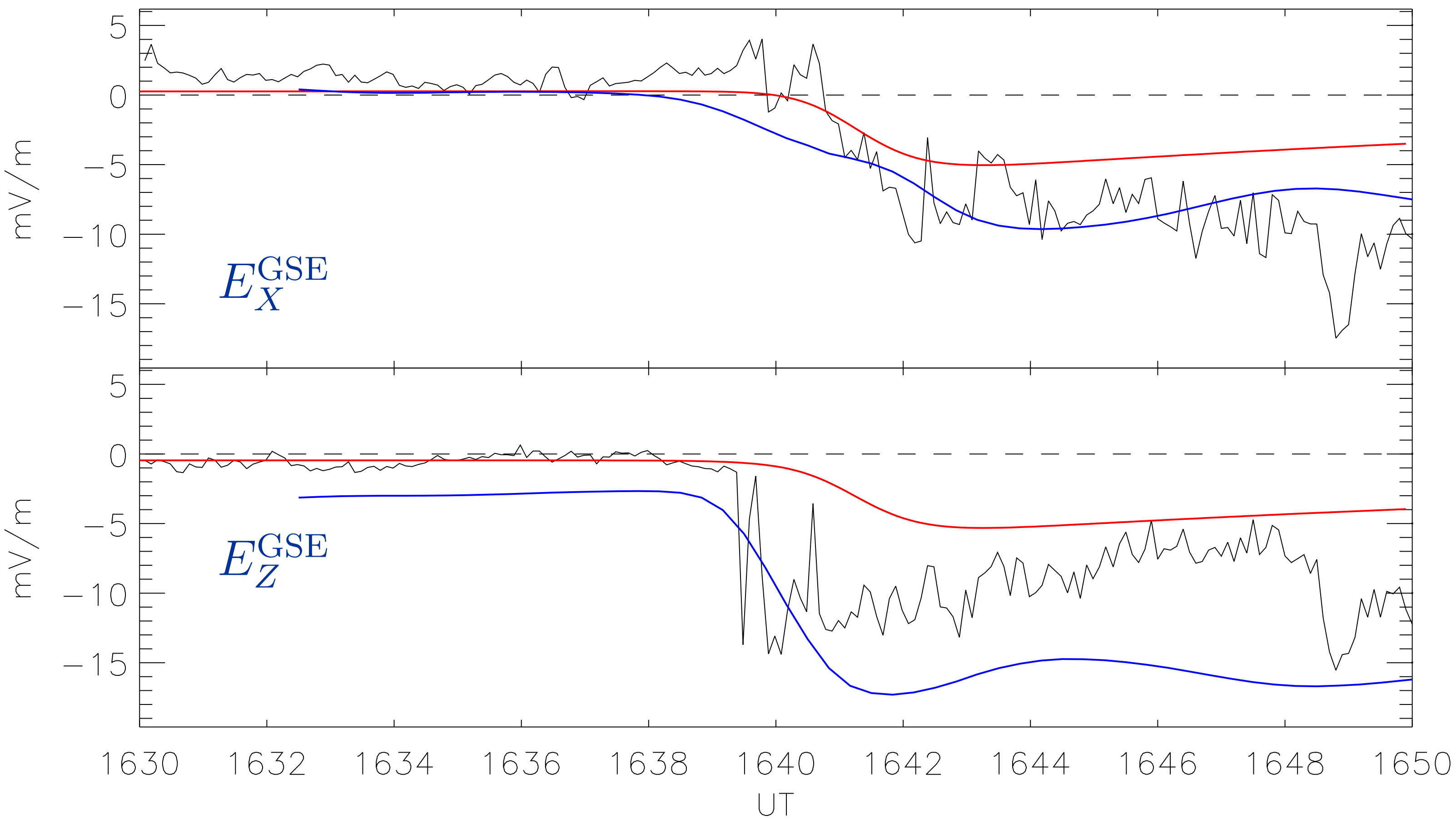
Electric Field Configurations (cont.)

- Induced electric field at 1640 UT
- Corresponds to the field lines motion
- Due to time-evolution of B_Y^{GSM}



Electric Field at Polar

Convection electic field, Polar (black), modelled (red), GUMICS (blue)



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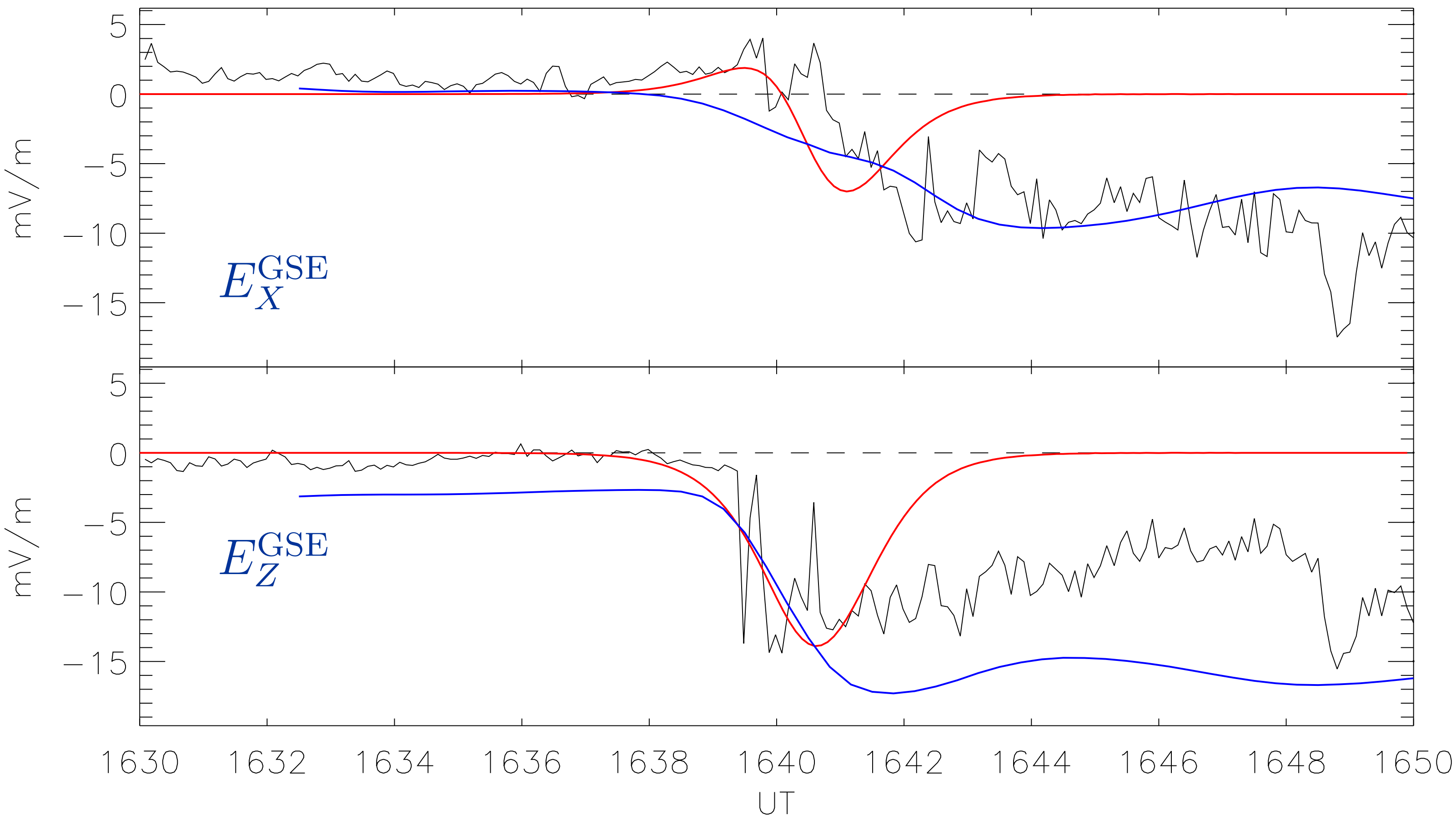
Electric Field at Polar (cont.)

Induced electic field, Polar (black), modelled (red), GUMICS (blue)



Electric Field at Polar (cont.)

Total electric field, Polar (black), modelled (red), GUMICS (blue)